Fiscal shocks and spillovers in a dynamic two-country model*

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Abstract

This paper examines the cross-border effects of domestic fiscal shocks on foreign economic activities by constructing a two-country general equilibrium model. The model yields two main results by comparing four alternative fiscal shocks: government spending, the capital income tax rate, the labor income tax rate, and the consumption tax rate. First, domestic fiscal shocks can generate sizable spillovers abroad. Second, once the size of each fiscal shock is normalized to achieve an equal change in government revenue, the spillover effects of different fiscal shocks on the foreign economic variables are qualitatively similar.

Keywords: government spending, tax rate, cross-border spillovers.

JEL Codes: E62, F40, H50.

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1 Introduction

This paper reconsiders the cross-border effects of fiscal policy. While the existing research focuses on the spillover effects of government spending, this paper compares the varying impacts of different instruments including government spending, the capital income tax rate, the labor income tax rate, and the consumption tax rate. The paper builds a two-country dynamic stochastic general equilibrium model based on Backus et al. (1994) (hereafter BKK).

Empirical studies find significant output spillovers from fiscal stimulus. For example, Beetsma et al. (2006) show that fiscal stimuli in Germany and France have non-negligible and positive output spillovers in all of the other 13 European countries in their sample. Auerbach and Gorodnichenko (2013) estimate cross-border output spillovers from government spending for a panel of OECD countries and find significant spillover effects of government spending on foreign output. The authors also find that the spillovers are stronger when the affected country is in a recession.[1] More recently, Dupor and McCrory (2018) find evidence of spillovers from fiscal policy between subregions of local labor markets.

Theoretical papers have also assessed the impact of fiscal spillovers. In Corsetti et al. (2010) and Corsetti and Müller (2013), the authors allow government spending to consolidate public debt and thus the government spending process displays a reversal feature, i.e., the current spending increase is accompanied by future cuts in spending. The foreign real long-term interest rate decreases in response to a domestic spending increase in their models, boosting foreign output. Devereux and Yu (2019) show that spillovers are affected by country size, openness, and the stance of monetary policy.

While Corsetti et al. (2010), Corsetti and Müller (2013), and Devereux and Yu (2019) study the cross-border spillovers of government spending shocks, this paper compares the different spillovers of government spending shocks and tax rate shocks. The paper is also related to Forni et al. (2009), Leeper, Plante, and Traum (2010), and Leeper, Walker, and Yang (2010). The authors also compare the varying impacts of different fiscal instruments, but their analysis is carried out within a closed economy environment and thus does not involve cross-border spillovers from fiscal policy.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents analytic results. Section 4 runs simulation and discusses in detail the spillovers of domestic fiscal shocks. Section 5 concludes.

2 Model structure

In order to analyze the spillover effects of fiscal shocks, the paper adds a fully-specified government sector in each country based on the BKK model. The model economy consists of two countries,

[1] This paper abstracts from state dependence. As summarized in Ramey (2019), whether the fiscal multiplier is state-dependent is still a disputed issue.

country 1 and country 2. Country 1 specializes in the production of good X and country 2 in the production of good Z. Labor and capital are internationally immobile. Consumption, investment, and government spending have both domestic and foreign content and use the same proportions of the two goods. They are composites of domestic and foreign goods as follows:

$$C_{1t} + I_{1t} + G_{1t} \equiv Q(X_{1t}, Z_{1t}) \tag{1}$$

$$C_{2t} + I_{2t} + G_{2t} \equiv Q(Z_{2t}, X_{2t}) \tag{2}$$

where $Q(X,Z) \equiv \left[\omega^{\frac{1}{\eta}} X^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} Z^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ is a CES aggregator. η measures the elasticity of substitution between domestic and foreign goods. The weight ω in Q determines the domestic and foreign content of domestic spending.

This is a model without money; all variables are real. Let q_{it}^X and q_{it}^Z denote the prices of good X and good Z, respectively, in period t in terms of the composite good in country i. X_{1t} and Z_{1t} are chosen to maximize

$$\left[\omega^{\frac{1}{\eta}} \left(X_{1t}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} \left(Z_{1t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - q_{1t}^X X_{1t} - q_{1t}^Z Z_{1t}$$

Similarly, X_{2t} and Z_{2t} are chosen to maximize

$$\left[\omega^{\frac{1}{\eta}} \left(Z_{2t}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}} \left(X_{2t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} - q_{2t}^Z Z_{2t} - q_{2t}^X X_{2t}$$

Thus the aggregate demand for the two goods is given by

$$X_{1t} = \omega \left(q_{1t}^X\right)^{-\eta} Q(X_{1t}, Z_{1t}) \tag{3}$$

$$Z_{1t} = (1-\omega) \left(q_{1t}^Z\right)^{-\eta} Q(X_{1t}, Z_{1t}) \tag{4}$$

$$Z_{2t} = \omega \left(q_{2t}^Z\right)^{-\eta} Q(Z_{2t}, X_{2t}) \tag{5}$$

$$X_{2t} = (1-\omega) \left(q_{2t}^X\right)^{-\eta} Q(Z_{2t}, X_{2t}) \tag{6}$$

where X_{2t} represents exports from country 1 to country 2, and Z_{1t} denotes imports into country 1.

2.1 Households

The representative household in each country i maximizes intertemporal utility characterized by functions of the form

$$E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\frac{C_{it}^{1-\gamma} - 1}{1-\gamma} - \theta \frac{L_{it}^{1+\kappa}}{1+\kappa} \right)$$

where C_{it} and L_{it} are consumption and hours worked, respectively, in country i. The household's budget constraint in country i is

$$(1+\tau_{it}^c)C_{it}+I_{it}+\Theta D_{it}=(1-\tau_{it}^l)q_{it}^{\Lambda}w_{it}L_{it}+(1-\tau_{it}^k)q_{it}^{\Lambda}R_{it}^kK_{i,t-1}+\Theta R_{i,t-1}D_{i,t-1}+(1-\Theta)\,T_{it} \quad (7)$$

where Λ is X for country 1 and Z for country 2. $\Theta \in \{0,1\}$ is an indicator function. In the analytical results section, Θ is set to equal 0 so that the model has minimum departure from the BKK model. Under this scenario, the household receives lump-sum transfers, T_{it}, from the government. In the simulation results section, Θ is equal to 1 and this corresponds to a real world with zero lump sum taxation. Under this scenario, the household purchases government bonds, D_{it}, and receives interest payments. We examine the dynamics of government debt and taxation when different fiscal shocks hit the economy.

Heathcote and Perri (2002) compares international business cycle models in which (1) markets are complete, (2) a single non-contingent bond is traded, and (3) there do not exist any markets for international trade in financial assets. The authors present the strength and shortcoming of using a financial autarky model compared to the other two models.[2] They also show that the equilibrium allocations in the bond model generally closely approximate those in the complete markets model.

The budget constraint in our baseline model assumes financial autarky. This assumption is maintained for analytic tractability. In Appendix D, we discuss a bond economy with trade in non-contingent bonds.

The household invests in physical capital. R_{it}^k is the capital rental rate. Consumption, labor income, and capital income are subject to taxation. τ_{it}^c, τ_{it}^l, and τ_{it}^k are tax rates on consumption, labor income, and capital income, respectively.[3]

The law of motion for capital is given by

$$K_{it}=(1-\delta)K_{i,t-1}+I_{it}-\frac{\phi}{2}\left(\frac{I_{it}}{K_{i,t-1}}-\delta\right)^2K_{i,t-1} \quad (8)$$

where δ is the depreciation rate. The last item captures the capital adjustment cost.[4] The household maximizes utility subject to its budget constraint and the law of motion for capital.

[2] They show that the financial autarky model reproduces many aspects of the data better than the other two models. However, it generates a linear relationship between the real exchange rate and the terms of trade, which is not in line with the data.

[3] Note that labor income and capital income are in units of the domestically produced good. Thus they are multiplied by the price of the domestic good. Consumption, investment, bond holdings, and transfers are expressed in terms of the composite good.

[4] International business cycle models use capital adjustment costs to reduce the volatility of investment in response to productivity shocks. BKK uses a time-to-build structure for capital formation as in Kydland and Prescott (1982). But convex capital adjustment costs have become more common since the publication of Baxter and Crucini (1995).

2.2 Firms

The representative firm in country 1 specializes in producing good X and the representative firm in country 2 specializes in producing good Z. They rent capital and labor from the households. Production functions in the two countries take the same form: $F(K, L) = K^{\alpha} L^{1-\alpha}$ where $\alpha \in [0, 1]$. The resource constraints in the two countries are

$$X_{1t} + X_{2t} = Y_{1t} = A_{1t} F(K_{1,t-1}, L_{1t}) \tag{9}$$

$$Z_{1t} + Z_{2t} = Y_{2t} = A_{2t} F(K_{2,t-1}, L_{2t}) \tag{10}$$

where Y_{it} represents production in country i in units of the domestically produced good, and X_{it} and Z_{it} are uses of the two goods in country i. Therefore, X_{it} and Z_{it} are intermediate goods and the composite goods are final goods.

The productivity shocks are assumed to follow the processes

$$\log(A_{1t}) = \rho_a \log(A_{1,t-1}) + \nu \log(A_{2,t-1}) + \varepsilon^a_{1t} \tag{11}$$

$$\log(A_{2t}) = \nu \log(A_{1,t-1}) + \rho_a \log(A_{2,t-1}) + \varepsilon^a_{2t} \tag{12}$$

where $\varepsilon^a_{it} \sim N(0, \sigma^2_a)$ and the contemporaneous correlation of the productivity shocks is $\lambda \in (0, 1)$. The parameter ν measures technology spillovers.

The representative firm in each country i maximizes its profit $Y_{it} - w_{it} L_{it} - R^k_{it} K_{i,t-1}$. Thus, wages and capital rental rates are $w_{it} = \frac{(1-\alpha)Y_{it}}{L_{it}}$ and $R^k_{it} = \frac{\alpha Y_{it}}{K_{i,t-1}}$, respectively.

2.3 Fiscal policy

The government's period budget constraint in each country i is

$$G_{it} + \Theta R_{i,t-1} D_{i,t-1} + (1-\Theta)\, T_{it} = \Theta D_{it} + \tau^k_{it} q^{\Lambda}_{it} R^k_{it} K_{i,t-1} + \tau^l_{it} q^{\Lambda}_{it} w_{it} L_{it} + \tau^c_{it} C_{it} \tag{13}$$

where Λ and Θ are the same as described in Section 2.1.

Let variables without a subscript t denote steady state values. Government spending, tax rates on capital income, labor income, and consumption evolve according to the following processes:

$$\log(G_{it}) = (1-\rho_g) \log(G_i) + \rho_G \log(G_{i,t-1}) + \sigma_g \varepsilon^G_{it} \tag{14}$$

$$\log(\tau^k_{it}) = (1-\rho_{\tau^k}) \log(\tau^k_i) + \rho_{\tau^k} \log(\tau^k_{i,t-1}) + \sigma_{\tau^k} \varepsilon^{\tau^k}_{it} \tag{15}$$

$$\log(\tau^l_{it}) = (1-\rho_{\tau^l}) \log(\tau^l_i) + \rho_{\tau^l} \log(\tau^l_{i,t-1}) + \sigma_{\tau^l} \varepsilon^{\tau^l}_{it} \tag{16}$$

$$\log(\tau^c_{it}) = (1-\rho_{\tau^c}) \log(\tau^c_i) + \rho_{\tau^c} \log(\tau^c_{i,t-1}) + \sigma_{\tau^c} \varepsilon^{\tau^c}_{it} \tag{17}$$

where each ε_{it} is an i.i.d. innovation with mean zero and standard deviation one.

Capital tax revenues, labor tax revenues, and consumption tax revenues are

$$T_{it}^k \equiv \tau_{it}^k q_{it}^{\Lambda} R_{it}^k K_{i,t-1}, \quad T_{it}^l \equiv \tau_{it}^l q_{it}^{\Lambda} w_{it} L_{it}, \quad \text{and} \quad T_{it}^c \equiv \tau_{it}^c C_{it}, \tag{18}$$

where Λ is X for country 1 and Z for country 2. Let REV_{it} denote total government revenue.

$$REV_{it} = T_{it}^k + T_{it}^l + T_{it}^c \tag{19}$$

Let "hat" variables denote log deviations from steady state. In a symmetric steady state, let $s_g \equiv \frac{G}{Y}$, $s_c \equiv \frac{C}{Y}$, $s_T \equiv \frac{T}{Y}$, and $s_d \equiv \frac{D}{Y}$.

Lemma 1. *When $\Theta = 0$, the relation between transfers and total government revenue is given by*

$$\hat{T}_{it} = -\frac{s_g}{s_T}\hat{G}_{it} + \frac{\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c}{s_T}\widehat{REV}_{it} \tag{20}$$

When $\Theta = 1$, the relations among debt, the real interest rate, and total government revenue are given by

$$\hat{D}_{it} = \frac{s_g}{s_d}\hat{G}_{it} + \frac{1}{\beta}\left(\hat{R}_{i,t-1} + \hat{D}_{i,t-1}\right) - \frac{\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c}{s_d}\widehat{REV}_{it} \tag{21}$$

$$\hat{R}_{it} = -\hat{D}_{it} - \frac{\beta s_g \rho_g}{s_d}\hat{G}_{it} + \beta E_t\left[\hat{D}_{i,t+1} + \frac{\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c}{s_d}\widehat{REV}_{i,t+1}\right] \tag{22}$$

Proof. In Appendix A. □

According to Lemma 1, if the three tax rate shocks are normalized so that each shock leads to an equal change in total government revenue, for example, $\widehat{REV}_{it} = \Omega$, then each shock will have the same effect on transfers when $\Theta = 0$, or on debt and the real interest rate when $\Theta = 1$.

Next, impact fiscal multipliers for output in period t, $GDP_{it} \equiv q_{it}^{\Lambda} Y_{it}$, are defined as

$$\textit{domestic multiplier} = \frac{\Delta GDP_{it}}{\Delta F_{it}} \tag{23}$$

$$\textit{spillover multiplier} = \frac{\Delta GDP_{jt}}{\Delta F_{it}} \tag{24}$$

where $j \neq i$ and $F_{it} \in \{G_{it}, T_{it}^k, T_{it}^l, T_{it}^c\}$.

2.4 National accounts

Recall that the optimal aggregate demand for the two goods in country 1 is given by $X_{1t} = \omega\left(q_{1t}^X\right)^{-\eta} Q(X_{1t}, Z_{1t})$ and $Z_{1t} = (1-\omega)\left(q_{1t}^Z\right)^{-\eta} Q(X_{1t}, Z_{1t})$. The aggregate demand for the two goods in country 2 is given by $Z_{2t} = \omega\left(q_{2t}^Z\right)^{-\eta} Q(Z_{2t}, X_{2t})$ and $X_{2t} = (1-\omega)\left(q_{2t}^X\right)^{-\eta} Q(Z_{2t}, X_{2t})$.

Using $C_{it} + I_{it} + G_{it} = Q_{it}$ and the resource constraints in the two countries, output can be decomposed as

$$GDP_{1t} \equiv \underbrace{q_{1t}^X Y_{1t}}_{\text{output}} = \underbrace{C_{1t} + I_{1t} + G_{1t}}_{\text{absorption}} + \underbrace{q_{1t}^X X_{2t}}_{\text{exports}} - \underbrace{q_{1t}^Z Z_{1t}}_{\text{imports}} \tag{25}$$

$$GDP_{2t} \equiv \underbrace{q_{2t}^Z Y_{2t}}_{\text{output}} = \underbrace{C_{2t} + I_{2t} + G_{2t}}_{\text{absorption}} + \underbrace{q_{2t}^Z Z_{1t}}_{\text{exports}} - \underbrace{q_{2t}^X X_{2t}}_{\text{imports}} \tag{26}$$

The trade balance is defined as the ratio of net exports to output:

$$TB_{1t} \equiv \left(X_{2t} - \frac{q_{1t}^Z}{q_{1t}^X} Z_{1t} \right) / Y_{1t}$$

$$TB_{2t} \equiv \left(Z_{1t} - \frac{q_{2t}^X}{q_{2t}^Z} X_{2t} \right) / Y_{2t}$$

Consistent with Heathcote and Perri (2002), in a model economy without internationally mobile financial assets, trade balance is equal to zero.

Lemma 2. *Trade is balanced in each country. That is,*

$$TB_{it} = 0 \tag{27}$$

Proof. In Appendix A. □

Suppose the law of one price holds. Real exchange rate is

$$RER_t = \frac{q_{1t}^X}{q_{2t}^X} = \frac{q_{1t}^Z}{q_{2t}^Z} \tag{28}$$

The following lemma describes the relations among real prices in the model economy.

Lemma 3. *Let* $s_{xz} \equiv \frac{X_1}{Y_1} = \frac{Z_2}{Y_2}$ *and* $\psi \equiv \omega^{\frac{1}{\eta}} (s_{xz})^{\frac{\eta-1}{\eta}}$. *The relationship between* $\hat{q}_{1t}^X$ *and the other three real prices,* $\hat{q}_{1t}^Z$, $\hat{q}_{2t}^X$, *and* $\hat{q}_{2t}^Z$, *are given by*

$$\hat{q}_{1t}^Z = -\frac{\psi}{1-\psi} \hat{q}_{1t}^X \tag{29}$$

$$\hat{q}_{2t}^X = \frac{\psi}{1-\psi} \hat{q}_{1t}^X \tag{30}$$

$$\hat{q}_{2t}^Z = -\hat{q}_{1t}^X \tag{31}$$

Proof. In Appendix A. □

The prices of a particular good are positively correlated across countries. Either within a country or across borders, the prices of good X and good Z are negatively correlated. Next, we look at the relationship between prices and production.

Lemma 4. *Let* $\zeta \equiv \frac{s_{xz}\eta-(2s_{xz}-1)(1-\psi+\eta\psi)}{(1-s_{xz})(1-\psi)}$. *The relationship between* $\hat{q}_{1t}^{X}$, $\hat{Y}_{1t}$, *and* $\hat{Y}_{2t}$ *is given by*

$$-\zeta \hat{q}_{1t}^{X} = \hat{Y}_{1t} - \hat{Y}_{2t} \tag{32}$$

Proof. In Appendix A. □

For a wide range of reasonable parameter combinations, ζ will be positive. An increase in the production level in a country will drive down the real price of the domestically produced good in the domestic country. The domestically produced good in the foreign country is also cheaper. Note that if the drop of price in the foreign country is larger than that in the domestic country, then there will be an appreciation of the real exchange rate.

3 Analytic results

In this section, Θ is set to equal zero. In each country, the representative household receives lump-sum transfers from the government and its budget constraint is

$$(1+\tau_{it}^{c})C_{it} + I_{it} = (1-\tau_{it}^{l})q_{it}^{\Lambda} w_{it} L_{it} + (1-\tau_{it}^{k})q_{it}^{\Lambda} R_{it}^{k} K_{i,t-1} + T_{it} \tag{33}$$

where Λ is X for country 1 and Z for country 2. For the governmnet in each country, spending and lump-sum transfers are financed through distortionary taxes. The government budget constraint is

$$G_{it} + T_{it} = \tau_{it}^{k} q_{it}^{\Lambda} R_{it}^{k} K_{i,t-1} + \tau_{it}^{l} q_{it}^{\Lambda} w_{it} L_{it} + \tau_{it}^{c} C_{it} \tag{34}$$

Equilibrium equations, steady state, and the log-linearized system are given in Appendix B. In order to obtain closed-form solutions, suppose capital is fixed at the initial value, does not depreciate, and investment is zero. For the sake of simplicity, also assume that productivity stays at the same level.

Proposition 1. *For* $j \neq i$, *output in country* i *is given by*

$$\begin{aligned}\widehat{GDP}_{it} \;=\; & \left(1-\frac{1}{\zeta}\right) A_p \hat{G}_{it} - \left(1-\frac{1}{\zeta}\right) B_p \hat{\tau}_{it}^{k} - \left(1-\frac{1}{\zeta}\right) C_p \hat{\tau}_{it}^{l} - \left(1-\frac{1}{\zeta}\right) D_p \hat{\tau}_{it}^{c} \\ & + \frac{1}{\zeta} A_p \hat{G}_{jt} - \frac{1}{\zeta} B_p \hat{\tau}_{jt}^{k} - \frac{1}{\zeta} C_p \hat{\tau}_{jt}^{l} - \frac{1}{\zeta} D_p \hat{\tau}_{jt}^{c}\end{aligned} \tag{35}$$

where $A_p = \frac{(1-\alpha)s_g}{\kappa+1}\left[1+\frac{(\gamma-s_c)\rho_g\xi}{(1-\rho_g)\gamma+\rho_g\xi s_c}\right]$, $B_p = \frac{(1-\alpha)\tau^k(\gamma-s_c)\rho_{\tau^k}\xi}{(\kappa+1)\left(1-\tau^k\right)\left[\left(1-\rho_{\tau^k}\right)\gamma+\rho_{\tau^k}\xi s_c\right]}$, $C_p = \frac{(1-\alpha)\tau^l}{(\kappa+1)\left(1-\tau^l\right)}$, *and* $D_p = \frac{(1-\alpha)\tau^c(1-\rho_{\tau^c}+\rho_{\tau^c}\xi)s_c}{(\kappa+1)(1+\tau^c)[(1-\rho_{\tau^c})\gamma+\rho_{\tau^c}\xi s_c]}$.

Proof. In Appendix A. □

Proposition 1 shows that output in each country is affected by both domestic fiscal shocks and foreign fiscal shocks. The coefficients $\frac{1}{\zeta}A_p$, $-\frac{1}{\zeta}B_p$, $-\frac{1}{\zeta}C_p$, and $-\frac{1}{\zeta}D_p$ in equation (35) reflect international effects of domestic policy. In line with the literature on cross-border effects of fiscal policy, these effects are called fiscal spillovers.[5]

Table 1: Domestic and spillover fiscal multipliers

	Spending	Capital tax	Labor tax	Consumption tax
Domestic multiplier	$\left(1-\frac{1}{\zeta}\right)A_p$	$-\frac{\left(1-\frac{1}{\zeta}\right)B_p}{1-\left(1-\frac{1}{\zeta}\right)B_p}$	$-\frac{\left(1-\frac{1}{\zeta}\right)C_p}{1-\left(1-\frac{1}{\zeta}\right)C_p}$	$-\frac{\left(1-\frac{1}{\zeta}\right)D_p}{1-\left[\frac{\tau^c}{(\gamma-s_c)(1+\tau^c)}-\frac{(\kappa+1)D_p}{(\gamma-s_c)(1-\alpha)}\right]}$
Spillover multiplier	$\frac{1}{\zeta}A_p$	$-\frac{\frac{1}{\zeta}B_p}{1-\left(1-\frac{1}{\zeta}\right)B_p}$	$-\frac{\frac{1}{\zeta}C_p}{1-\left(1-\frac{1}{\zeta}\right)C_p}$	$-\frac{\frac{1}{\zeta}D_p}{1-\left[\frac{\tau^c}{(\gamma-s_c)(1+\tau^c)}-\frac{(\kappa+1)D_p}{(\gamma-s_c)(1-\alpha)}\right]}$

Notes: $\zeta \equiv \frac{s_{xz}\eta-(2s_{xz}-1)(1-\psi+\eta\psi)}{(1-s_{xz})(1-\psi)}$, $s_{xz} \equiv \frac{X_1}{Y_1} = \frac{Z_2}{Y_2}$, $\psi \equiv \omega^{\frac{1}{\eta}}\left(s_{xz}\right)^{\frac{\eta-1}{\eta}}$, $A_p = \frac{(1-\alpha)s_g}{\kappa+1}\left[1+\frac{(\gamma-s_c)\rho_g\xi}{\left(1-\rho_g\right)\gamma+\rho_g\xi s_c}\right]$, $B_p = \frac{(1-\alpha)\tau^k(\gamma-s_c)\rho_{\tau^k}\xi}{(\kappa+1)\left(1-\tau^k\right)\left[\left(1-\rho_{\tau^k}\right)\gamma+\rho_{\tau^k}\xi s_c\right]}$, $C_p = \frac{(1-\alpha)\tau^l}{(\kappa+1)\left(1-\tau^l\right)}$, and $D_p = \frac{(1-\alpha)\tau^c(1-\rho_{\tau^c}+\rho_{\tau^c}\xi)s_c}{(\kappa+1)(1+\tau^c)[(1-\rho_{\tau^c})\gamma+\rho_{\tau^c}\xi s_c]}$.

Table 1 summarizes the various multipliers calculated from the model economy. Details of the calculation appear in Appendix B. Now specific parameters are plugged into the model in order for us to examine the fiscal multipliers.

The discount factor, β, is set to equal 0.99. The risk aversion parameter, γ, takes a standard value of 2. The labor supply power, κ, is set to equal 0. The utility weight of work parameter, θ, is set to 1.94 so that hours worked in steady state is 0.33. The home bias parameter, ω, and the steady state share of domestically produced intermediate goods in final goods production, s_{xz}, are both set to equal 0.8. The capital share in production, α, is set equal to 0.3. The parameters related to technology are taken from BKK: $\rho_a = 0.906$, $\sigma_a = 0.00852$, $\nu = 0.088$, and $\lambda = 0.258$.

Since all the data series involved in the feedback rules are available, including real government spending and tax rates on capital income, labor income, and consumption, the fiscal rules are estimated outside the model: $\rho_g = 0.9717$, $\rho_{\tau^k} = 0.9598$, $\rho_{\tau^l} = 0.9771$, $\rho_{\tau^c} = 0.9827$, $\sigma_g = 0.0141$, $\sigma_{\tau^k} = 0.0229$, $\sigma_{\tau^l} = 0.0244$, and $\sigma_{\tau^c} = 0.0211$. The steady state spending to output ratio and tax rates are set to equal their sample means: $s_g = 0.1754$, $\tau^k = 0.3762$, $\tau^l = 0.2175$, and $\tau^c = 0.0929$. Appendix E describes the data sources and construction.

Suppose the home bias parameter, ω, and the share of locally produced intermediate goods in composite goods production, s_{xz}, are equal ($\omega = s_{xz}$). Then $\psi \equiv \omega^{\frac{1}{\eta}}\left(s_{xz}\right)^{1-\frac{1}{\eta}} = \omega$ and $\zeta \equiv \frac{1-2\omega(1-\eta)}{1-\omega}$. If $\eta \geq 0.5$, then $\zeta \geq 1$. Given the values of the other parameters (either standard or estimated from data), we have that A_p, B_p, C_p, and D_p are all less than one. Therefore, if $\omega = s_{xz}$ and $\eta \geq 0.5$, then spending multipliers (both domestic and spillover) are positive; tax multipliers

[5] See, for example, Corsetti et al. (2010), Auerbach and Gorodnichenko (2013), and Devereux and Yu (2019).

(both domestic and spillover) are negative.

Figure 1 plots domestic and spillover fiscal multipliers as we vary η, the elasticity of substitution between domestic and foreign goods. Panels (a) through (d) plot domestic multipliers. Panels (e) through (h) plot spillover multipliers. Recall from Lemma 4 that $-\zeta \hat{q}_{1t}^X = \hat{Y}_{1t} - \hat{Y}_{2t}$ where $\psi \equiv \omega^{\frac{1}{\eta}} (s_{xz})^{\frac{\eta-1}{\eta}}$ and $\zeta \equiv \frac{s_{xz}\eta-(2s_{xz}-1)(1-\psi+\eta\psi)}{(1-s_{xz})(1-\psi)}$. Given our parameter choices, $\omega = 0.8$, $s_{xz} = 0.8$, and $\eta \in [0.5, 1.5]$, ζ ranges from 1 to 9. Due to home bias, an expansionary domestic shock drives up domestic production to a larger extent, compared with foreign production. According to the result in Lemma 4, this makes domestically produced goods relatively cheaper and foreign goods more expensive. As the elasticity of substitution between domestic and foreign goods increase, it is more tempting for domestic agents to lean towards cheaper domestic goods. Correspondingly, domestic multipliers are higher and spillover multipliers are lower (in terms of absolute value).

As emphasized in Ramey (2019), the size of fiscal multipliers will depend on features that characterize the model economy, including exchange rate regime and openness. For instance, Ilzetzki et al. (2013) estimate government spending multipliers to vary from 0.15 on impact to 1.4 in the long run for fixed exchange rate regimes and between 0.14 to -0.69 for flexible exchange rate regimes. In open economies, some of the expansionary effect of fiscal stimulus falls on foreign goods, so multipliers are smaller compared to closed economies. The impact government spending multiplier estimated in Ilzetzki et al. (2013) is 0.61 for closed economies and -0.077 for open economies. The size of multipliers in this paper is consistent with exiting literature on open economy fiscal multipliers.

The main takeaway from Table 1 and Figure 1 is that while open economy multipliers are small, the model is able to generate sizeable spillovers, consistent with empirical findings in Auerbach and Gorodnichenko (2013).

Next, the size of each shock is normalized so that each shock leads to the same percentage change in domestic total government revenue.

Proposition 2. *Suppose that following a domestic fiscal shock, domestic total government revenue increases by Ω percent. That is,*

$$\widehat{REV}_{it} = \Omega \tag{36}$$

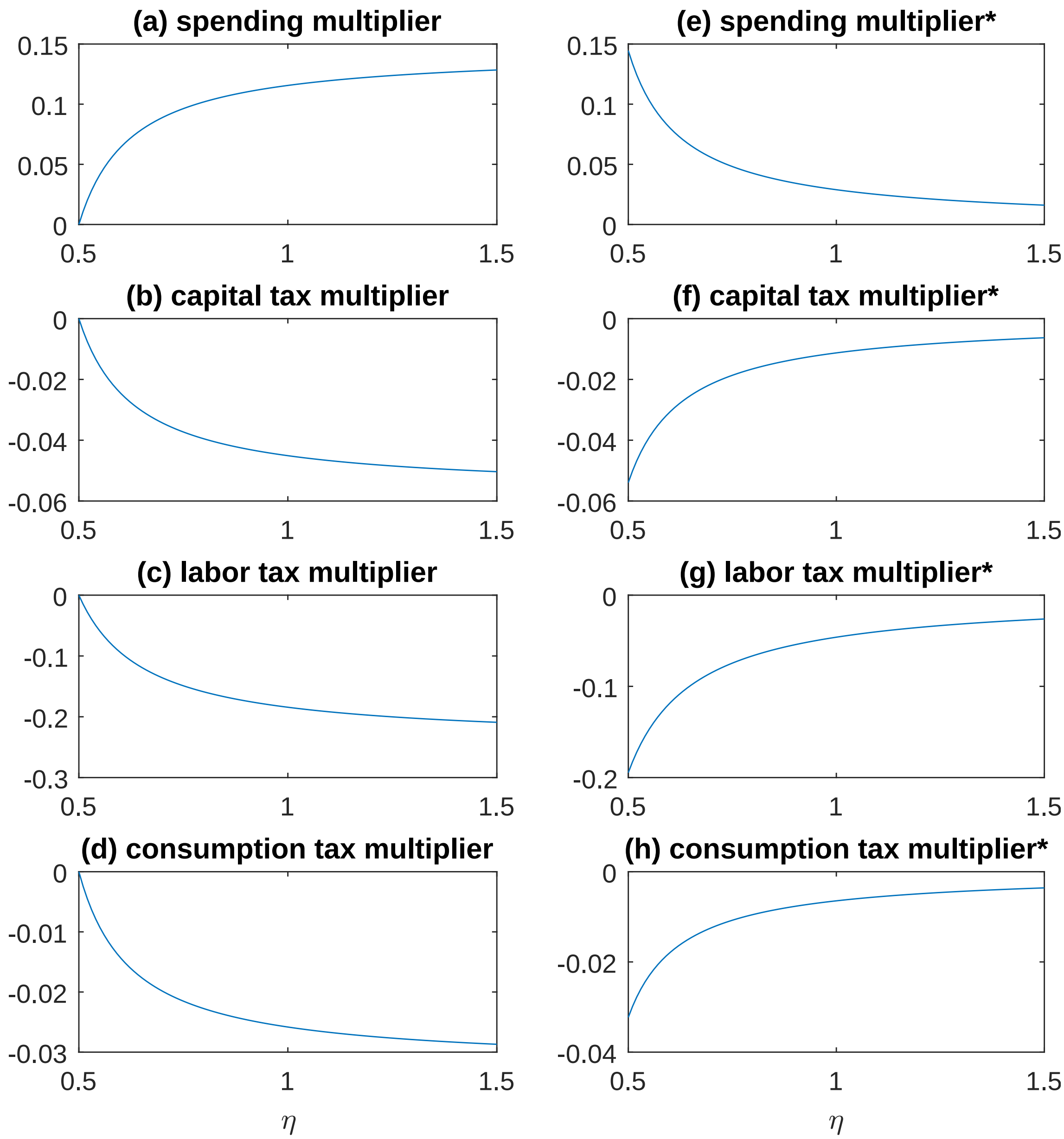
The required size of each fiscal shock is

$$\hat{G}_{it} = \frac{\Gamma}{\Psi s_g + \Phi A_p} \tag{37}$$

$$\hat{\tau}_{it}^k = \frac{\Gamma}{\tau^k \alpha - \Phi B_p} \tag{38}$$

$$\hat{\tau}_{it}^l = \frac{\Gamma}{\tau^l (1-\alpha) - \Psi \frac{\tau^l}{1-\tau^l} - \Phi C_p} \tag{39}$$

Figure 1: Domestic and spillover multipliers, varying η



Notes: η = elasticity of substitution between domestic and foreign goods. (a)-(d) plot impact domestic fiscal multipliers. (e)-(h) plot impact spillover multipliers.

$$\hat{\tau}^c_{it} = \frac{\Gamma}{\tau^c s_c - \Psi \frac{\tau^c}{1+\tau^c} - \Phi D_p} \tag{40}$$

where $\Gamma \equiv \left[\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c\right] \Omega$, $\Phi \equiv \left[\tau^k\alpha + \tau^l(1-\alpha)\right]\left(1-\frac{1}{\zeta}\right) - \frac{\tau^c s_c(\kappa+1)}{(\gamma - s_c)(1-\alpha)}$, *and* $\Psi \equiv \frac{\tau^c s_c}{\gamma - s_c}$.

Proof. In Appendix A. □

Column (1) in Figure 2 plots the required size of each domestic fiscal shock to achieve an 1% increase in domestic total government revenue, i.e., $\Omega = 1$, as we vary η. Note that

$$\left[\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c\right] \widehat{REV}_{it} = \tau^k\alpha\hat{\tau}^k_{it} + \tau^l(1-\alpha)\hat{\tau}^l_{it} + \tau^c s_c \hat{\tau}^c_{it} + \tau^c s_c \hat{C}_{it} + \left[\tau^k\alpha + \tau^l(1-\alpha)\right]\widehat{GDP}_{it} \tag{41}$$

An increase in government spending boosts output and leads to increased government revenue. As η increases, the stimulative effect of spending becomes stronger as households lean towards cheaper domestic goods. Therefore, the required size of spending shrinks while maintaining the same change of government revenue. In contrast, increased tax rates dampen output and this dampening effect gets stronger as η increases. Therefore, in order to maintain the same change of government revenue, the required size of spending rises as η increases.

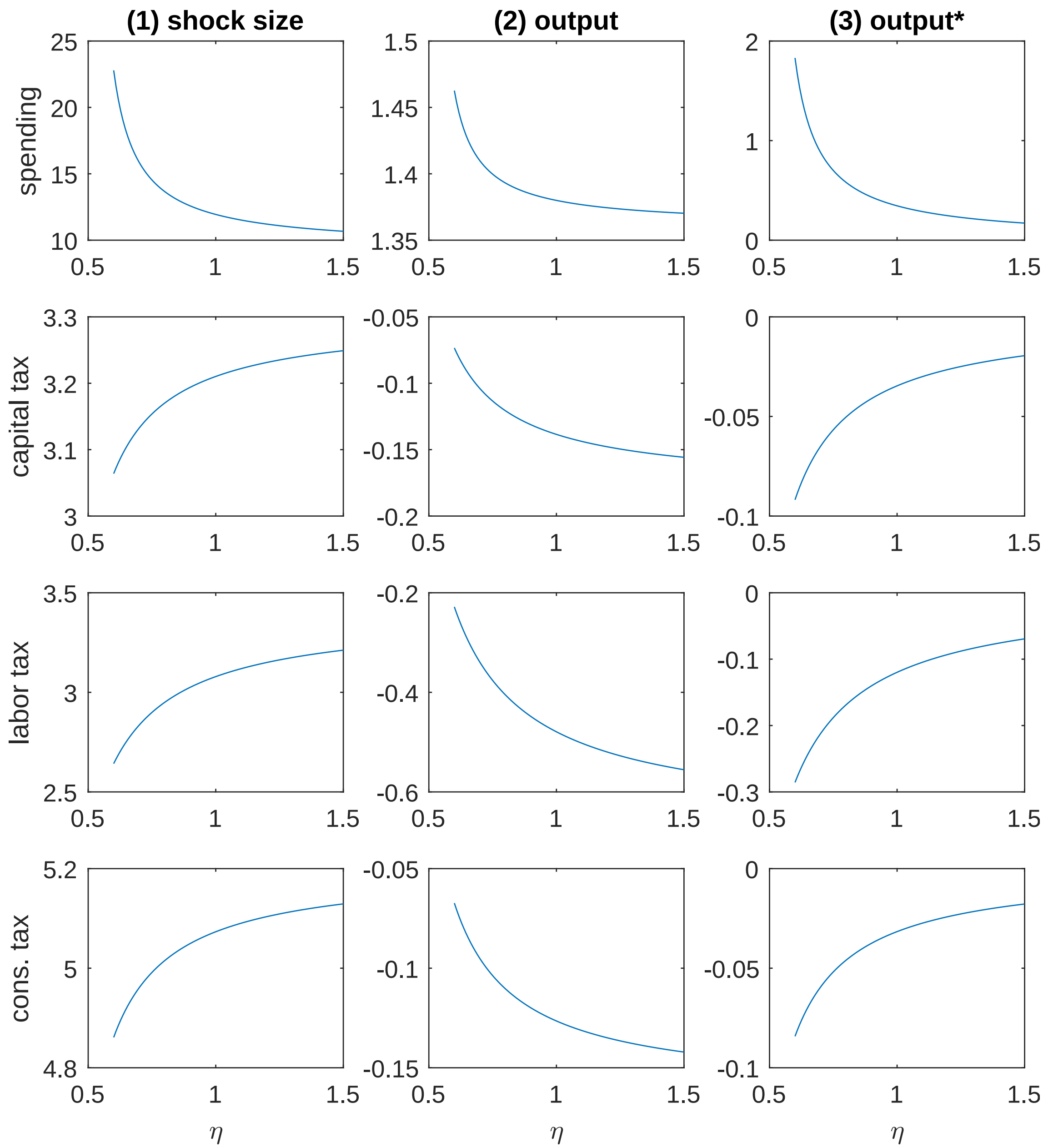
Table 2: Domestic and foreign output in response to normalized fiscal shocks

	Spending	Capital tax	Labor tax	Consumption tax
Domestic output	$\frac{\left(1-\frac{1}{\zeta}\right)A_p\Gamma}{\Psi s_g + \Phi A_p}$	$-\frac{\left(1-\frac{1}{\zeta}\right)B_p\Gamma}{\tau^k\alpha - \Phi B_p}$	$-\frac{\left(1-\frac{1}{\zeta}\right)C_p\Gamma}{\tau^l(1-\alpha) - \Psi\frac{\tau^l}{1-\tau^l} - \Phi C_p}$	$-\frac{\left(1-\frac{1}{\zeta}\right)D_p\Gamma}{\tau^c s_c - \Psi\frac{\tau^c}{1+\tau^c} - \Phi D_p}$
Foreign output	$\frac{\frac{1}{\zeta}A_p\Gamma}{\Psi s_g + \Phi A_p}$	$-\frac{\frac{1}{\zeta}B_p\Gamma}{\tau^k\alpha - \Phi B_p}$	$-\frac{\frac{1}{\zeta}C_p\Gamma}{\tau^l(1-\alpha) - \Psi\frac{\tau^l}{1-\tau^l} - \Phi C_p}$	$-\frac{\frac{1}{\zeta}D_p\Gamma}{\tau^c s_c - \Psi\frac{\tau^c}{1+\tau^c} - \Phi D_p}$

Notes: $\zeta \equiv \frac{s_{xz}\eta - (2s_{xz}-1)(1-\psi+\eta\psi)}{(1-s_{xz})(1-\psi)}$, $s_{xz} \equiv \frac{X_1}{Y_1} = \frac{Z_2}{Y_2}$, $\psi \equiv \omega^{\frac{1}{\eta}}(s_{xz})^{\frac{\eta-1}{\eta}}$, $A_p = \frac{(1-\alpha)s_g}{\kappa+1}\left[1 + \frac{(\gamma-s_c)\rho_g\xi}{(1-\rho_g)\gamma + \rho_g\xi s_c}\right]$, $B_p = \frac{(1-\alpha)\tau^k(\gamma-s_c)\rho_{\tau^k}\xi}{(\kappa+1)(1-\tau^k)\left[(1-\rho_{\tau^k})\gamma + \rho_{\tau^k}\xi s_c\right]}$, $C_p = \frac{(1-\alpha)\tau^l}{(\kappa+1)(1-\tau^l)}$, $D_p = \frac{(1-\alpha)\tau^c(1-\rho_{\tau^c}+\rho_{\tau^c}\xi)s_c}{(\kappa+1)(1+\tau^c)[(1-\rho_{\tau^c})\gamma + \rho_{\tau^c}\xi s_c]}$, $\Gamma \equiv \left[\tau^k\alpha + \tau^l(1-\alpha) + \tau^c s_c\right]\Omega$, $\Phi \equiv \left[\tau^k\alpha + \tau^l(1-\alpha)\right]\left(1-\frac{1}{\zeta}\right) - \frac{\tau^c s_c(\kappa+1)}{(\gamma-s_c)(1-\alpha)}$, $\Psi \equiv \frac{\tau^c s_c}{\gamma - s_c}$.

Table 2 summarizes the responses of domestic and foreign output following each normalized fiscal shock. Set $\Omega = 1$. Columns (2) and (3) in Figure 2 plot output responses as we vary η, the elasticity of substitution between domestic and foreign goods. For tax rates, the patterns of output responses are similar to those of multipliers in Figure 1. However, the response of domestic output becomes smaller as η increases. The fall in domestic output response is due to a decreasing size of the spending shock.

Figure 2: Shock size and output responses, varying η



Notes: η = elasticity of substitution between domestic and foreign goods. Column (1) plots the size of each domestic fiscal shock to achieve an 1% increase in domestic government revenue. Column (2) plots domestic output responses to each fiscal shock. Column (3) plots foreign output responses to each fiscal shock.

4 Simulation

In this section, Θ is set to equal one. In each country, the representative household purchases government bonds and its budget constraint is

$$(1+\tau_{it}^{c})C_{it}+I_{it}+D_{it}=(1-\tau_{it}^{l})q_{it}^{\Lambda}w_{it}L_{it}+(1-\tau_{it}^{k})q_{it}^{\Lambda}R_{it}^{k}K_{i,t-1}+R_{i,t-1}D_{i,t-1} \tag{42}$$

where Λ is X for country 1 and Z for country 2. Government spending is financed through distortionary taxes and the issuance of debt. The government budget constraint is

$$G_{it}+R_{i,t-1}D_{i,t-1}=D_{it}+\tau_{it}^{k}q_{it}^{\Lambda}R_{it}^{k}K_{i,t-1}+\tau_{it}^{l}q_{it}^{\Lambda}w_{it}L_{it}+\tau_{it}^{c}C_{it} \tag{43}$$

Equilibrium equations, steady state, and the log-linearized system are given in Appendix C. By allowing for the dynamics of capital and bonds, we lose analytic tractability and use simulation to explore the effects of various fiscal shocks.

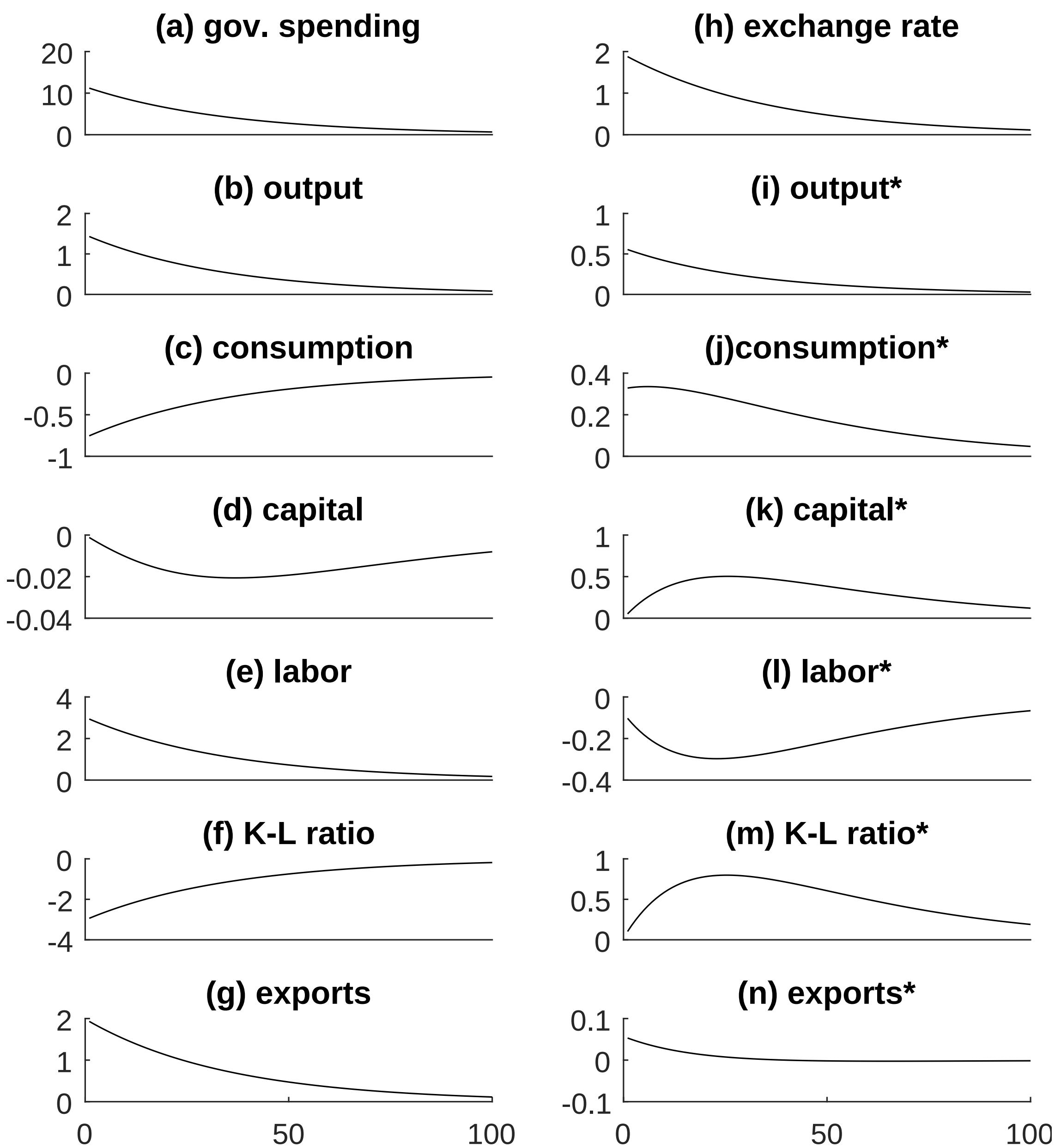
Following an expansionary fiscal shock, the domestic levels of consumption and investment will change. Because these components of output are an aggregate of domestic and foreign products, consumers will directly spend part of their income abroad, changing the demand for foreign imports and thus foreign production. Moreover, if the real exchange rate appreciates for domestic consumers, the foreign country's international competitiveness is enhanced, thus boosting its exports to the domestic economy.

Figures 3-6 plot the impulse responses following an exogenous change in each fiscal instrument in the domestic country. The size of each fiscal shock is normalized so that total government revenue of the domestic country increases by 1% on impact. Variables are measured in percentage deviation from the steady state. Foreign variables are indicated with an asterisk (*). Note that in equilibrium, trade is balanced in each country. So the impulse response of exports and that of imports are the same.

Figure 3 shows how key domestic and foreign variables adjust to the domestic government spending impulse. Consistent with neoclassical growth models, a spending increase stimulates domestic production but leads to a drop in private consumption. The domestic government funds its spending by issuing government bonds and collecting taxes. Because of the negative wealth effect, domestic consumers increase their labor supply while at the same time cutting their consumption. In the meantime, private investment is crowded out because of a higher real interest rate. Correspondingly, the capital to labor input ratio decreases.

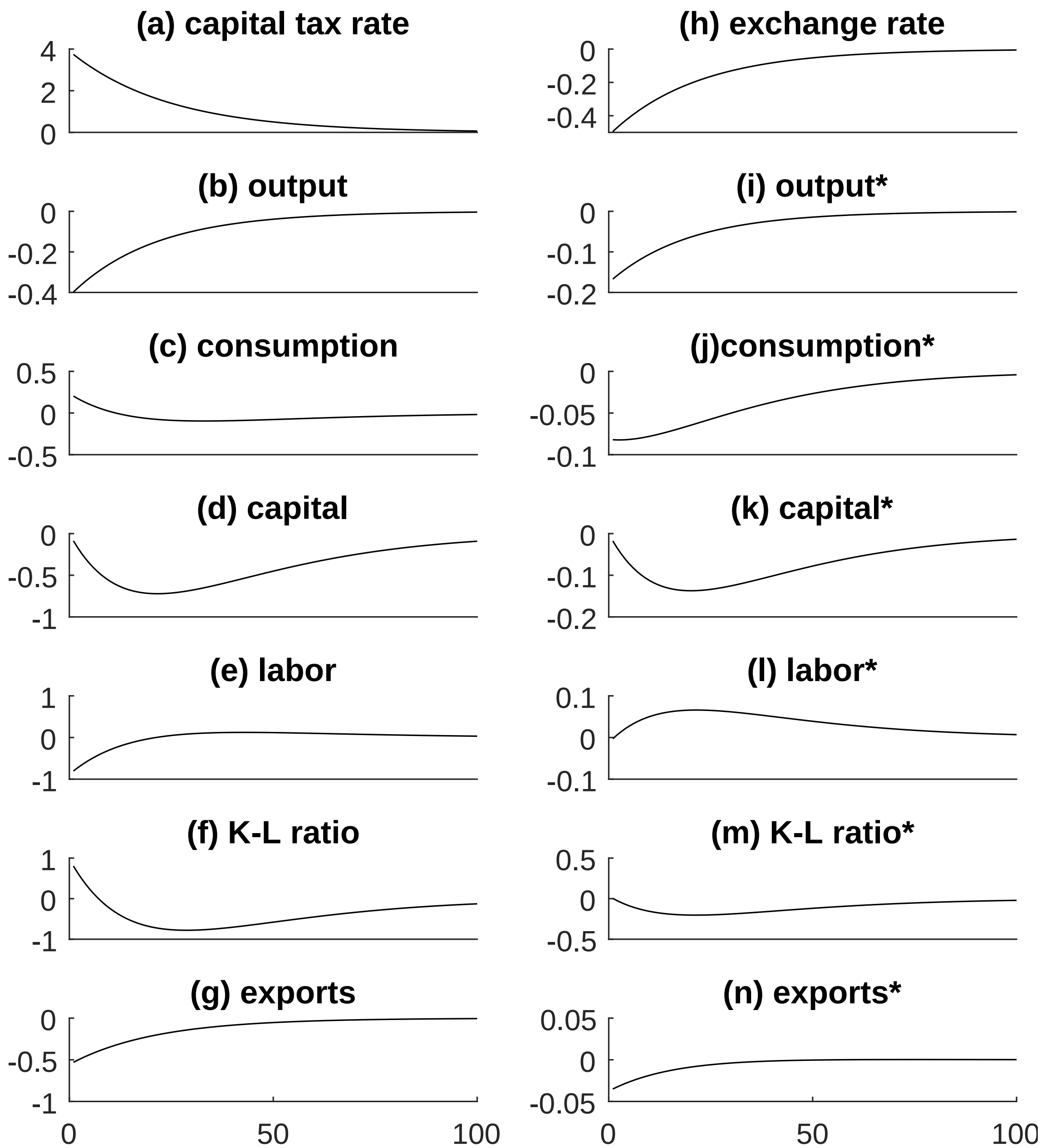
The increase in government spending promotes international trade, raising exports and imports. The real exchange rate appreciates in the domestic country, meaning that the price drop of the domestically produced goods is even larger in the foreign country than that in the domestic country. As the real exchange rate appreciates for domestic consumers, the foreign country's international competitiveness is enhanced, thus boosting its exports to the domestic economy. Higher demand

Figure 3: Responses to a domestic government spending shock



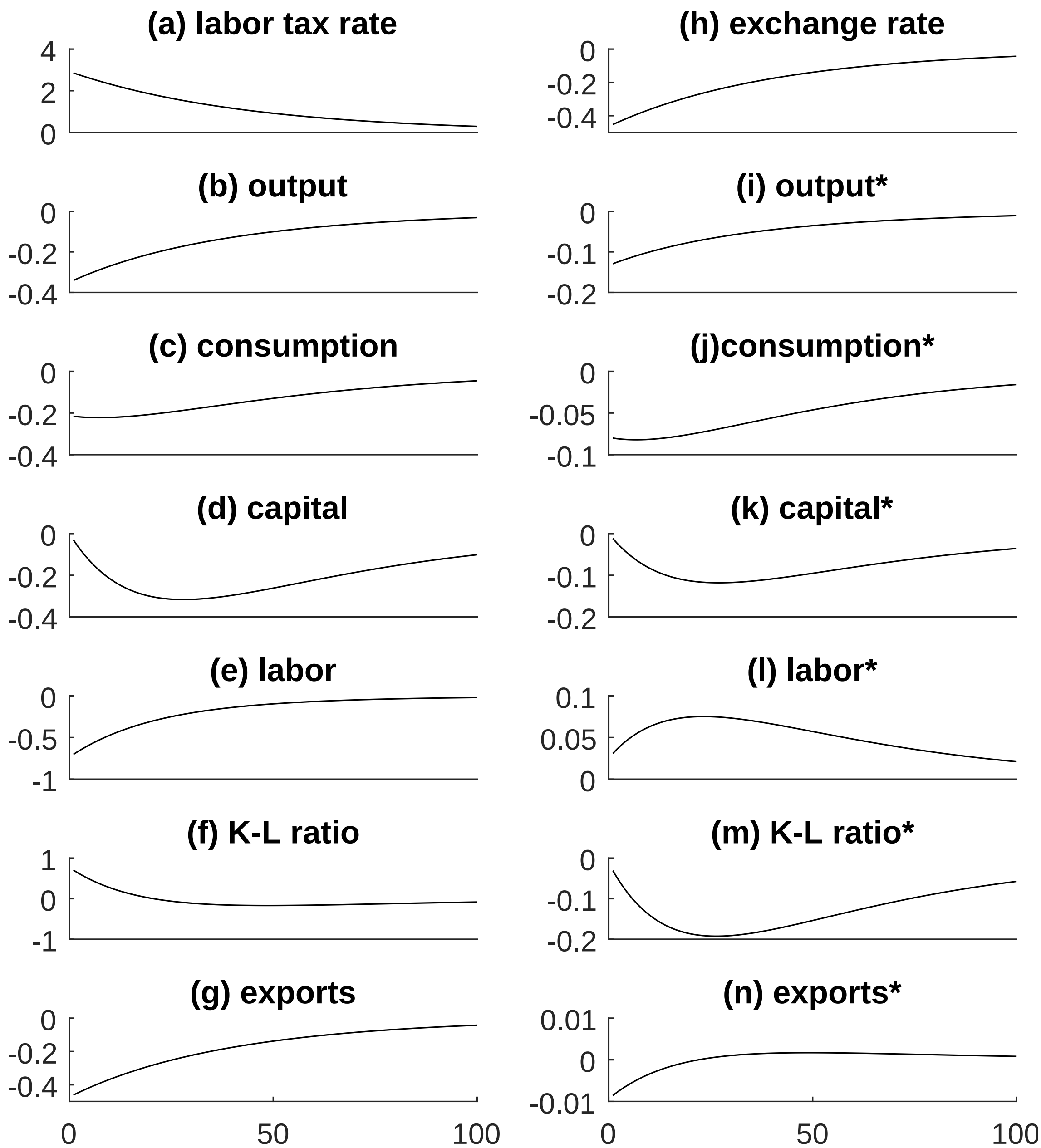
Notes: Responses are measured in percentage deviations from the steady-state level. Horizontal axes measure time in quarters.

Figure 4: Responses to a domestic capital tax rate shock



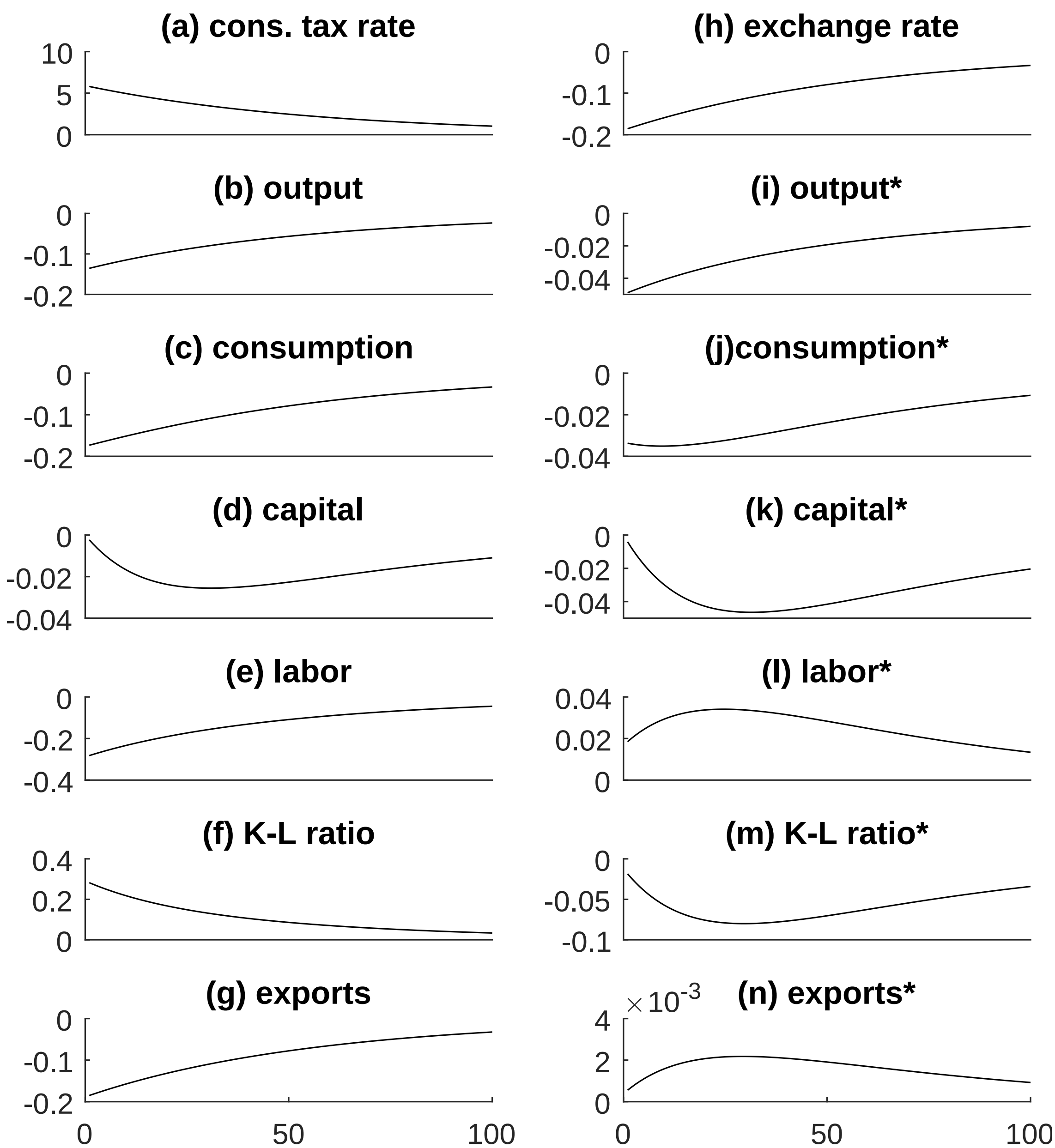
Notes: Responses are measured in percentage deviations from the steady-state level. Horizontal axes measure time in quarters.

Figure 5: Responses to a domestic labor tax rate shock



Notes: Responses are measured in percentage deviations from the steady-state level. Horizontal axes measure time in quarters.

Figure 6: Responses to a domestic consumption tax rate shock



Notes: Responses are measured in percentage deviations from the steady-state level. Horizontal axes measure time in quarters.

from the domestic country leads to an increase of output and investment in the foreign country. Foreign households enjoy more consumption and leisure time. Correspondingly, the capital to labor input ratio increases. Overall, cross-country spillovers are significant, i.e., foreign variables respond in magnitude comparable to that of domestic variables.

Figures 4, 5, and 6 document impulse responses to contractionary domestic tax shocks. Each foreign variable behaves similarly across different tax shocks. And each foreign variable has an opposite sign compared to its counterpart under an expansionary spending shock. Therefore, when different fiscal shocks are normalized into equal revenue terms, their spillover effects on foreign economic activities are qualitatively similar.

Most domestic variables also behave similarly. One exception is domestic consumption. The capital tax rate shock leads households to save less and consume more. The labor income tax rate shock decreases households' disposable income and they reduce consumption correspondingly. For the consumption tax shock, households decrease their consumption due to a higher effective price of consumption goods.

Two results should be highlighted. First, the spillover effects of fiscal shocks are sizable. Second, when different fiscal shocks are normalized to equal revenue terms, the spillover effects from each fiscal instrument exhibit remarkable similarity.

5 Conclusion

This paper contributes to the debate on international spillovers of fiscal policy by comparing the effects of different fiscal instruments. It shows that once normalized into equal revenue terms, each fiscal shock has qualitatively similar effects on foreign economic variables. This paper serves as a theoretical benchmark for generating sizable cross-border spillovers of fiscal policy. The current model is kept deliberately simple so that results are easily interpretable. The model abstracts from nominal rigidities and monetary policies. It would be important to explore how the interplay between monetary and fiscal policies influences cross-border spillovers of fiscal stimulus. This direction is left for future research.

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Appendix A Proofs of lemmas and propositions

A.1 Proof of Lemma 1

Proof. Rewrite the government's budget constraint

$$G_{it} + \Theta R_{i,t-1} D_{i,t-1} + (1-\Theta) T_{it} = \Theta D_{it} + REV_{it}$$

Let $s_g \equiv \frac{G}{Y}$, $s_c \equiv \frac{C}{Y}$, $s_T \equiv \frac{T}{Y}$, and $s_d \equiv \frac{D}{Y}$. Log-linearize

$$s_g \hat{G}_{it} + \Theta \frac{1}{\beta} s_d \hat{R}_{i,t-1} + \Theta \frac{1}{\beta} s_d \hat{D}_{i,t-1} + (1-\Theta) s_T \hat{T}_{it} = \Theta s_d \hat{D}_{it} + \left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \widehat{REV}_{it}$$

When $\Theta = 0$

$$s_g \hat{G}_{it} + s_T \hat{T}_{it} = \left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \widehat{REV}_{it}$$

$$\hat{T}_{it} = -\frac{s_g}{s_T} \hat{G}_{it} + \frac{\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c}{s_T} \widehat{REV}_{it}$$

When $\Theta = 1$

$$s_g \hat{G}_{it} + \frac{1}{\beta} s_d \hat{R}_{i,t-1} + \frac{1}{\beta} s_d \hat{D}_{i,t-1} = s_d \hat{D}_{it} + \left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \widehat{REV}_{it}$$

$$\hat{D}_{it} = \frac{s_g}{s_d} \hat{G}_{it} + \frac{1}{\beta} \left(\hat{R}_{i,t-1} + \hat{D}_{i,t-1}\right) - \frac{\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c}{s_d} \widehat{REV}_{it}$$

$$\hat{R}_{it} = -\hat{D}_{it} - \frac{\beta s_g \rho_g}{s_d} \hat{G}_{it} + \beta E_t \left[\hat{D}_{i,t+1} + \frac{\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c}{s_d} \widehat{REV}_{i,t+1}\right]$$

□

A.2 Proof of Lemma 2

Proof. Combining the household's and the government's budget constraints in each country, we obtain

$$C_{1t} + I_{1t} + G_{1t} = q^X_{1t} w_{1t} L_{1t} + q^X_{1t} R^k_{1t} K_{1,t-1}$$

$$C_{2t} + I_{2t} + G_{2t} = q^Z_{2t} w_{2t} L_{2t} + q^Z_{2t} R^k_{2t} K_{2,t-1}$$

Noting that $w_{it} L_{it} = (1-\alpha) Y_{it}$ and $R^k_{it} K_{i,t-1} = \alpha Y_{it}$, we have

$$C_{1t} + I_{1t} + G_{1t} = q^X_{1t} Y_{1t} \quad \text{and} \quad C_{2t} + I_{2t} + G_{2t} = q^Z_{2t} Y_{2t}$$

Recall that

$$q^X_{1t} Y_{1t} = (C_{1t} + I_{1t} + G_{1t}) + q^X_{1t} X_{2t} - q^Z_{1t} Z_{1t}$$

$$q_{2t}^{Z}Y_{2t} = (C_{2t} + I_{2t} + G_{2t}) + q_{2t}^{Z}Z_{1t} - q_{2t}^{X}X_{2t}$$

Therefore,

$$q_{1t}^{X}X_{2t} = q_{1t}^{Z}Z_{1t} \quad \text{and} \quad q_{2t}^{Z}Z_{1t} = q_{2t}^{X}X_{2t}$$

And hence

$$TB_{1t} \equiv \left(X_{2t} - \frac{q_{1t}^{Z}}{q_{1t}^{X}}Z_{1t}\right)/Y_{1t} = 0 \quad \text{and} \quad TB_{2t} \equiv \left(Z_{1t} - \frac{q_{2t}^{X}}{q_{2t}^{Z}}X_{2t}\right)/Y_{2t} = 0$$

□

A.3 Proof of Lemma 3

Proof. First, we prove $\hat{q}_{1t}^{Z} = -\frac{\psi}{1-\psi}\hat{q}_{1t}^{X}$ and $\hat{q}_{2t}^{X} = -\frac{\psi}{1-\psi}\hat{q}_{2t}^{Z}$. Recall that

$$Q(X_{1t}, Z_{1t}) \equiv C_{1t} + I_{1t} + G_{1t} = q_{1t}^{X}Y_{1t} \quad \text{and} \quad Q(Z_{2t}, X_{2t}) \equiv C_{2t} + I_{2t} + G_{2t} = q_{2t}^{Z}Y_{2t}$$

Moreover,

$$X_{1t} = \omega\left(q_{1t}^{X}\right)^{-\eta}Q(X_{1t}, Z_{1t}) \quad \text{and} \quad Z_{1t} = (1-\omega)\left(q_{1t}^{Z}\right)^{-\eta}Q(X_{1t}, Z_{1t})$$

$$Z_{2t} = \omega\left(q_{2t}^{Z}\right)^{-\eta}Q(Z_{2t}, X_{2t}) \quad \text{and} \quad X_{2t} = (1-\omega)\left(q_{2t}^{X}\right)^{-\eta}Q(Z_{2t}, X_{2t})$$

Combining the above equations, we obtain

$$X_{1t} = \omega\left(q_{1t}^{X}\right)^{1-\eta}Y_{1t} \quad \text{and} \quad Z_{1t} = (1-\omega)\left(q_{1t}^{Z}\right)^{-\eta}q_{1t}^{X}Y_{1t}$$

$$Z_{2t} = \omega\left(q_{2t}^{Z}\right)^{1-\eta}Y_{2t} \quad \text{and} \quad X_{2t} = (1-\omega)\left(q_{2t}^{X}\right)^{-\eta}q_{2t}^{Z}Y_{2t}$$

Log-linearize the above equations

$$\hat{X}_{1t} = (1-\eta)\,\hat{q}_{1t}^{X} + \hat{Y}_{1t} \quad \text{and} \quad \hat{Z}_{1t} = -\eta\hat{q}_{1t}^{Z} + \hat{q}_{1t}^{X} + \hat{Y}_{1t}$$

$$\hat{Z}_{2t} = (1-\eta)\,\hat{q}_{2t}^{Z} + \hat{Y}_{2t} \quad \text{and} \quad \hat{X}_{2t} = -\eta\hat{q}_{2t}^{X} + \hat{q}_{2t}^{Z} + \hat{Y}_{2t}$$

Let $s_{xz} \equiv \frac{X_1}{Y_1} = \frac{Z_2}{Y_2}$. From the resource constraints $X_{1t}+X_{2t} = Y_{1t}$ and $Z_{1t}+Z_{2t} = Y_{2t}$, we have that $\frac{X_2}{Y_1} = \frac{Z_1}{Y_2} = 1 - s_{xz}$. Let $\psi \equiv \omega^{\frac{1}{\eta}}\left(s_{xz}\right)^{\frac{\eta-1}{\eta}}$. From $q_{1t}^{X}Y_{1t} = \left[\omega^{\frac{1}{\eta}}\left(X_{1t}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}}\left(Z_{1t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ and $\left(q_{2t}^{Z}\right)\left(Y_{2t}\right) = \left[\omega^{\frac{1}{\eta}}\left(Z_{2t}\right)^{\frac{\eta-1}{\eta}} + (1-\omega)^{\frac{1}{\eta}}\left(X_{2t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$, we get

$$\hat{q}_{1t}^{X} + \hat{Y}_{1t} = \psi\hat{X}_{1t} + (1-\psi)\,\hat{Z}_{1t} \quad \text{and} \quad \hat{q}_{2t}^{Z} + \hat{Y}_{2t} = \psi\hat{Z}_{2t} + (1-\psi)\,\hat{X}_{2t}$$

Plugging $\hat{X}_{1t}$, $\hat{Z}_{1t}$, $\hat{Z}_{2t}$, and $\hat{X}_{2t}$ into the above equations, we obtain

$$\psi \hat{q}^X_{1t} + (1-\psi)\,\hat{q}^Z_{1t} = 0 \quad \text{and} \quad \psi \hat{q}^Z_{2t} + (1-\psi)\,\hat{q}^X_{2t} = 0$$

Therefore,

$$\hat{q}^Z_{1t} = -\frac{\psi}{1-\psi}\hat{q}^X_{1t} \quad \text{and} \quad \hat{q}^X_{2t} = -\frac{\psi}{1-\psi}\hat{q}^Z_{2t}$$

Next, we prove $\hat{q}^Z_{2t} = -\hat{q}^X_{1t}$. Recall the definition of the real exchange rate $RER_t = \frac{q^X_{1t}}{q^X_{2t}} = \frac{q^Z_{1t}}{q^Z_{2t}}$ Log-linearization yields

$$\hat{q}^X_{1t} + \hat{q}^Z_{2t} = \hat{q}^Z_{1t} + \hat{q}^X_{2t}$$

Plug $\hat{q}^Z_{1t} = -\frac{\psi}{1-\psi}\hat{q}^X_{1t}$ and $\hat{q}^X_{2t} = -\frac{\psi}{1-\psi}\hat{q}^Z_{2t}$ into the above equation

$$\hat{q}^X_{1t} + \hat{q}^Z_{2t} = -\frac{\psi}{1-\psi}\hat{q}^X_{1t} - \frac{\psi}{1-\psi}\hat{q}^Z_{2t}$$

Rearrange

$$\hat{q}^X_{1t} = -\hat{q}^Z_{2t}$$

□

A.4 Proof of Lemma 4

Proof. Log-linearize the resource constraint in country 1

$$s_{xz}\hat{X}_{1t} + (1-s_{xz})\,\hat{X}_{2t} = \hat{Y}_{1t}$$

Plug $\hat{X}_{1t} = (1-\eta)\,\hat{q}^X_{1t} + \hat{Y}_{1t}$, $\hat{X}_{2t} = -\eta\hat{q}^X_{2t} + \hat{q}^Z_{2t} + \hat{Y}_{2t}$, $\hat{q}^X_{2t} = \frac{\psi}{1-\psi}\hat{q}^X_{1t}$, and $\hat{q}^Z_{2t} = -\hat{q}^X_{1t}$ into the above equation

$$s_{xz}\,(1-\eta)\,\hat{q}^X_{1t} + s_{xz}\hat{Y}_{1t} - (1-s_{xz})\,\eta\frac{\psi}{1-\psi}\hat{q}^X_{1t} - (1-s_{xz})\,\hat{q}^X_{1t} + (1-s_{xz})\,\hat{Y}_{2t} = \hat{Y}_{1t}$$

Rearrange

$$\frac{(2s_{xz}-1)\,(1-\psi+\eta\psi) - s_{xz}\eta}{(1-s_{xz})\,(1-\psi)}\hat{q}^X_{1t} = \hat{Y}_{1t} - \hat{Y}_{2t}$$

Note that the same result can be obtained if we use equations from country 2. □

A.5 Proof of Proposition 1

Proof. Given that capital and productivity are fixed, we have that $\hat{K}_{it} = 0$, $\hat{A}_{it} = 0$, and $\hat{L}_{it} = \frac{1}{1-\alpha}\hat{Y}_{it}$. The log-linearized euqations become

$$s_g\hat{G}_{it} + s_T\hat{T}_{it} = \alpha\tau^k\hat{\tau}^k_{it} + (1-\alpha)\tau^l\hat{\tau}^l_{it} + \tau^c s_c\hat{\tau}^c_{it} + \left[\alpha\tau^k + (1-\alpha)\tau^l\right]\left(s_c\hat{C}_{it} + s_g\hat{G}_{it}\right) + \tau^c s_c\hat{C}_{it} \quad (44)$$

$$\hat{C}_{it} = \frac{s_g}{\gamma - s_c}\hat{G}_{it} - \frac{1}{\gamma - s_c}\frac{\tau^l}{1-\tau^l}\hat{\tau}^l_{it} - \frac{1}{\gamma - s_c}\frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{it} - \frac{\kappa+1}{\gamma - s_c}\frac{1}{1-\alpha}\hat{Y}_{it} \quad (45)$$

$$\gamma\hat{C}_{it} + (1-\rho_{\tau^c})\frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{it} + \xi s_g\rho_g\hat{G}_{it} = (\gamma - \xi s_c)\, E_t\hat{C}_{i,t+1} + \xi\frac{\tau^k}{1-\tau^k}\rho_{\tau^k}\hat{\tau}^k_{it} \quad (46)$$

The variables can be solved with the method of undetermined coefficients. Guess

$$\hat{Y}_{1t} = A_p\hat{G}_{1t} - B_p\hat{\tau}^k_{1t} - C_p\hat{\tau}^l_{1t} - D_p\hat{\tau}^c_{1t} + G_p\hat{G}_{2t} - H_p\hat{\tau}^k_{2t} - I_p\hat{\tau}^l_{2t} - J_p\hat{\tau}^c_{2t}$$

$$\hat{Y}_{2t} = A_p\hat{G}_{2t} - B_p\hat{\tau}^k_{2t} - C_p\hat{\tau}^l_{2t} - D_p\hat{\tau}^c_{2t} + G_p\hat{G}_{1t} - H_p\hat{\tau}^k_{1t} - I_p\hat{\tau}^l_{1t} - J_p\hat{\tau}^c_{1t}$$

From equations (31) and (32), $\hat{q}^X_{1t}$ and $\hat{q}^Z_{2t}$ can be written as functions of $\hat{G}_{it}$, $\hat{\tau}^k_{it}$, $\hat{\tau}^l_{it}$, $\hat{\tau}^c_{it}$ where $i = 1, 2$. By plugging $\hat{Y}_{1t}$ and $\hat{Y}_{2t}$ into (45), we could obtain $\hat{C}_{it}$. Update $\hat{C}_{it}$ by one period and obtain $E_t\hat{C}_{i,t+1}$. Plugging $\hat{C}_{it}$ and $E_t\hat{C}_{i,t+1}$ into (46) and matching coefficients, we get

$$A_p = \frac{(1-\alpha)\, s_g}{\kappa+1}\left[1 + \frac{(\gamma - s_c)\,\rho_g\xi}{(1-\rho_g)\,\gamma + \rho_g\xi s_c}\right]$$

$$B_p = \frac{(1-\alpha)\,\tau^k\,(\gamma - s_c)\,\rho_{\tau^k}\xi}{(\kappa+1)\,(1-\tau^k)\,[(1-\rho_{\tau^k})\,\gamma + \rho_{\tau^k}\xi s_c]}$$

$$C_p = \frac{(1-\alpha)\,\tau^l}{(\kappa+1)\,(1-\tau^l)}$$

$$D_p = \frac{(1-\alpha)\,\tau^c\,(1-\rho_{\tau^c} + \rho_{\tau^c}\xi)\, s_c}{(\kappa+1)\,(1+\tau^c)\,[(1-\rho_{\tau^c})\,\gamma + \rho_{\tau^c}\xi s_c]}$$

$$G_p = H_p = I_p = J_p = 0$$

Therefore,

$$\hat{Y}_{it} = A_p\hat{G}_{it} - B_p\hat{\tau}^k_{it} - C_p\hat{\tau}^l_{it} - D_p\hat{\tau}^c_{it}$$

$$\hat{q}^A_{it} = \frac{A_p}{-\zeta}\hat{G}_{it} - \frac{B_p}{-\zeta}\hat{\tau}^k_{it} - \frac{C_p}{-\zeta}\hat{\tau}^l_{it} - \frac{D_p}{-\zeta}\hat{\tau}^c_{it} + \frac{-A_p}{-\zeta}\hat{G}_{jt} - \frac{-B_p}{-\zeta}\hat{\tau}^k_{jt} - \frac{-C_p}{-\zeta}\hat{\tau}^l_{jt} - \frac{-D_p}{-\zeta}\hat{\tau}^c_{jt}$$

and

$$
\begin{aligned}
\hat{q}_{it}^{\Lambda} + \hat{Y}_{it} \;&=\; \left(1-\frac{1}{\zeta}\right) A_p \hat{G}_{it} - \left(1-\frac{1}{\zeta}\right) B_p \hat{\tau}_{it}^{k} - \left(1-\frac{1}{\zeta}\right) C_p \hat{\tau}_{it}^{l} - \left(1-\frac{1}{\zeta}\right) D_p \hat{\tau}_{it}^{c} \\
&\quad + \frac{1}{\zeta} A_p \hat{G}_{jt} - \frac{1}{\zeta} B_p \hat{\tau}_{jt}^{k} - \frac{1}{\zeta} C_p \hat{\tau}_{jt}^{l} - \frac{1}{\zeta} D_p \hat{\tau}_{jt}^{c}
\end{aligned}
$$

□

A.6 Proof of Proposition 2 and calculation in Table 2

Let $\hat{G}_{jt} = \hat{\tau}_{1t}^{k} = \hat{\tau}_{2t}^{k} = \hat{\tau}_{1t}^{l} = \hat{\tau}_{2t}^{l} = \hat{\tau}_{1t}^{c} = \hat{\tau}_{2t}^{c} = 0$. Then

$$\widehat{GDP}_{it} = \left(1-\frac{1}{\zeta}\right) A_p \hat{G}_{it}$$

$$\widehat{GDP}_{jt} = \frac{1}{\zeta} A_p \hat{G}_{it}$$

$$\hat{C}_{it} = \left[\frac{s_g}{\gamma - s_c} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} A_p\right] \hat{G}_{it}$$

Choose $\hat{G}_{it}$ so that $\widehat{REV}_{it} = \varOmega$.

$$\varOmega = \frac{\tau^k \alpha + \tau^l (1-\alpha)}{\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c} \widehat{GDP}_{it} + \frac{\tau^c s_c}{\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c} \hat{C}_{it}$$

Plug $\widehat{GDP}_{it}$ and $\hat{C}_{it}$ into the above equation and solve $\hat{G}_{it}$

$$\hat{G}_{it} = \frac{\left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \varOmega}{\left[\tau^k \alpha + \tau^l (1-\alpha)\right] \left(1-\frac{1}{\zeta}\right) A_p + \tau^c s_c \left[\frac{s_g}{\gamma - s_c} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} A_p\right]}$$

Hence,

$$\widehat{GDP}_{it} = \frac{\left(1-\frac{1}{\zeta}\right) A_p \left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \varOmega}{\tau^c s_c \frac{s_g}{\gamma - s_c} + \left\{\left[\tau^k \alpha + \tau^l (1-\alpha)\right] \left(1-\frac{1}{\zeta}\right) - \frac{\tau^c s_c (\kappa+1)}{(\gamma - s_c)(1-\alpha)}\right\} A_p}$$

$$\widehat{GDP}_{jt} = \frac{\frac{1}{\zeta} A_p \left[\tau^k \alpha + \tau^l (1-\alpha) + \tau^c s_c\right] \varOmega}{\tau^c s_c \frac{s_g}{\gamma - s_c} + \left\{\left[\tau^k \alpha + \tau^l (1-\alpha)\right] \left(1-\frac{1}{\zeta}\right) - \frac{\tau^c s_c (\kappa+1)}{(\gamma - s_c)(1-\alpha)}\right\} A_p}$$

Similarly, let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}_{jt}^{k} = \hat{\tau}_{1t}^{l} = \hat{\tau}_{2t}^{l} = \hat{\tau}_{1t}^{c} = \hat{\tau}_{2t}^{c} = 0$ and choose $\hat{\tau}_{it}^{k}$ so that $\widehat{REV}_{it} = \varOmega$. Let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}_{1t}^{k} = \hat{\tau}_{2t}^{k} = \hat{\tau}_{jt}^{l} = \hat{\tau}_{1t}^{c} = \hat{\tau}_{2t}^{c} = 0$ and choose $\hat{\tau}_{it}^{l}$ so that $\widehat{REV}_{it} = \varOmega$. Let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}_{1t}^{k} = \hat{\tau}_{2t}^{k} = \hat{\tau}_{1t}^{l} = \hat{\tau}_{2t}^{l} = \hat{\tau}_{jt}^{c} = 0$ and choose $\hat{\tau}_{it}^{c}$ so that $\widehat{REV}_{it} = \varOmega$. The procedures are the same and hence omitted to save space.

Appendix B Solving the model in Section 3

B.1 Equilibrium conditions

$$\theta(1+\tau_{it}^c)L_{it}^{\kappa+\alpha}C_{it}^{\gamma} = (1-\tau_{it}^l)q_{it}^{\Lambda}(1-\alpha)A_{it}K_{1,t-1}^{\alpha} \tag{47}$$

$$\frac{C_{it}^{-\gamma}}{(1+\tau_{it}^c)} = \beta E_t \frac{C_{i,t+1}^{-\gamma}}{(1+\tau_{i,t+1}^c)}\left\{(1-\tau_{i,t+1}^k)q_{i,t+1}^{\Lambda}R_{i,t+1}^k + (1-\delta)\right\} \tag{48}$$

$$Y_{it} = A_{it}K_{1,t-1}^{\alpha}\left(L_{it}\right)^{1-\alpha} \tag{49}$$

$$G_{it} + T_{it} = \tau_{it}^k q_{it}^{\Lambda}\alpha Y_{it} + \tau_{it}^l q_{it}^{\Lambda}(1-\alpha)Y_{it} + \tau_{it}^c C_{it} \tag{50}$$

$$R_{it}^k = \frac{\alpha Y_{it}}{K_{1,t-1}} \tag{51}$$

Equations (47)-(51) along with the law of motion for capital, (8), the productivity shock processes, (11) and (12), and the fiscal policy rules, (14)-(17), define a competitive equilibrium.

B.2 Computation of steady state

Let $s_g \equiv \frac{G}{Y}$. In a symmetric steady state, q^X and q^Z are normalized to equal 1. From equation (48)

$$R^k = \frac{1/\beta - (1-\delta)}{(1-\tau^k)}$$

From equation (51)

$$s_k \equiv \frac{K}{Y} = \frac{\alpha\left(1-\tau^k\right)}{1/\beta - (1-\delta)}$$

From $I = \delta K$ and $C + I + G = Y$

$$s_c \equiv \frac{C}{Y} = 1 - s_g - \delta s_k$$

From equation (50)

$$s_T \equiv \frac{T}{Y} = \alpha\tau^k + (1-\alpha)\tau^l + \tau^c s_c - s_g$$

Combining equations (47) and (49), we get

$$\theta = \frac{(1-\tau^l)(1-\alpha)A^{\frac{1-\gamma}{1-\alpha}}\left(s_k\right)^{\frac{\alpha(1-\gamma)}{1-\alpha}}}{(1+\tau^c)\left(s_c\right)^{\gamma}L^{\kappa+\gamma}}$$

θ is chosen so that $L = 0.33$. Y can be solved from equation (49)

$$Y = A^{\frac{1}{1-\alpha}}\left(s_k\right)^{\frac{\alpha}{1-\alpha}}L$$

After we solve Y, it is trivial to solve C, K, I, and T.

B.3 Log-linearized system

Replace $\hat{R}^k_{1,t+1}$ by $\hat{Y}_{1,t+1} - \hat{K}_{1,t}$. The log-linearized system consists of equations (31), (32), and

$$\hat{A}_{1t} = \rho_a \hat{A}_{1,t-1} + \nu \hat{A}_{2,t-1} + \varepsilon^a_{1t} \tag{52}$$

$$\hat{A}_{2t} = \nu \hat{A}_{1,t-1} + \rho_a \hat{A}_{2,t-1} + \varepsilon^a_{2t} \tag{53}$$

$$\hat{G}_{it} = \rho_G \hat{G}_{i,t-1} + \sigma_G \varepsilon^G_{it} \tag{54}$$

$$\hat{\tau}^k_{it} = \rho_{\tau^k} \hat{\tau}^k_{i,t-1} + \sigma_{\tau^k} \varepsilon^{\tau^k}_{it} \tag{55}$$

$$\hat{\tau}^l_{it} = \rho_{\tau^l} \hat{\tau}^l_{i,t-1} + \sigma_{\tau^l} \varepsilon^{\tau^l}_{it} \tag{56}$$

$$\hat{\tau}^c_{it} = \rho_{\tau^c} \hat{\tau}^c_{i,t-1} + \sigma_{\tau^c} \varepsilon^{\tau^c}_{it} \tag{57}$$

$$\hat{Y}_{it} = \hat{A}_{it} + \alpha \hat{K}_{i,t-1} + (1-\alpha)\hat{L}_{it} \tag{58}$$

$$(\kappa+1)\hat{L}_{it} + \gamma \hat{C}_{it} + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{it} = \left(\hat{q}^{\Lambda}_{it} + \hat{Y}_{it}\right) - \frac{\tau^l}{1-\tau^l}\hat{\tau}^l_{it} \tag{59}$$

$$s_g \hat{G}_{it} + s_T \hat{T}_{it} = \alpha \tau^k \hat{\tau}^k_{it} + (1-\alpha)\tau^l \hat{\tau}^l_{it} + \tau^c s_c \hat{\tau}^c_{it} + (s_g + s_z - \tau^c s_c)\left(\hat{q}^{\Lambda}_{it} + \hat{Y}_{it}\right) + \tau^c s_c \hat{C}_{it} \tag{60}$$

$$\gamma \hat{C}_{it} + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{it} = E_t\left[\gamma \hat{C}_{i,t+1} + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{i,t+1} + \xi \frac{\tau^k}{1-\tau^k}\hat{\tau}^k_{i,t+1} - \xi\left(\hat{q}^X_{i,t+1} + \hat{Y}_{i,t+1}\right) + \xi \hat{K}_{i,t}\right] \tag{61}$$

$$\hat{K}_{it} = (1-\delta)\hat{K}_{i,t-1} - \frac{s_c}{s_k}\hat{C}_{it} - \frac{s_g}{s_k}\hat{G}_{it} + \frac{1}{s_k}\left(\hat{q}^{\Lambda}_{it} + \hat{Y}_{it}\right) \tag{62}$$

B.4 Calculating fiscal multipliers

Recall that

$$\hat{Y}_{it} = A_p \hat{G}_{it} - B_p \hat{\tau}^k_{it} - C_p \hat{\tau}^l_{it} - D_p \hat{\tau}^c_{it}$$

$$\begin{aligned}\widehat{GDP}_{it} \quad = \quad & \left(1-\frac{1}{\zeta}\right) A_p \hat{G}_{it} - \left(1-\frac{1}{\zeta}\right) B_p \hat{\tau}^k_{it} - \left(1-\frac{1}{\zeta}\right) C_p \hat{\tau}^l_{it} - \left(1-\frac{1}{\zeta}\right) D_p \hat{\tau}^c_{it} \\ & + \frac{1}{\zeta} A_p \hat{G}_{jt} - \frac{1}{\zeta} B_p \hat{\tau}^k_{jt} - \frac{1}{\zeta} C_p \hat{\tau}^l_{jt} - \frac{1}{\zeta} D_p \hat{\tau}^c_{jt}\end{aligned}$$

$$
\begin{aligned}
\hat{C}_{it} &= \left[\frac{s_g}{\gamma - s_c} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} A_p\right] \hat{G}_{it} + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} B_p \hat{\tau}_{it}^k \\
&\quad - \left[\frac{\tau^l}{(\gamma - s_c)(1-\tau^l)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} C_p\right] \hat{\tau}_{it}^l - \left[\frac{\tau^c}{(\gamma - s_c)(1+\tau^c)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} D_p\right] \hat{\tau}_{it}^c \\
&\quad - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} G_p \hat{G}_{jt} + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} H_p \hat{\tau}_{jt}^k + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} I_p \hat{\tau}_{jt}^l + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} J_p \hat{\tau}_{jt}^c
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\hat{T}_{it}^k &= \left(1 - \frac{1}{\zeta}\right) A_p \hat{G}_{it} + \left[1 - \left(1 - \frac{1}{\zeta}\right) B_p\right] \hat{\tau}_{it}^k - \left(1 - \frac{1}{\zeta}\right) C_p \hat{\tau}_{it}^l - \left(1 - \frac{1}{\zeta}\right) D_p \hat{\tau}_{it}^c \\
&\quad + \frac{1}{\zeta} A_p \hat{G}_{jt} - \frac{1}{\zeta} B_p \hat{\tau}_{jt}^k - \frac{1}{\zeta} C_p \hat{\tau}_{jt}^l - \frac{1}{\zeta} D_p \hat{\tau}_{jt}^c
\end{aligned}
$$

$$
\begin{aligned}
\hat{T}_{it}^l &= \left(1 - \frac{1}{\zeta}\right) A_p \hat{G}_{it} - \left(1 - \frac{1}{\zeta}\right) B_p \hat{\tau}_{it}^k + \left[1 - \left(1 - \frac{1}{\zeta}\right) C_p\right] \hat{\tau}_{it}^l - \left(1 - \frac{1}{\zeta}\right) D_p \hat{\tau}_{it}^c \\
&\quad + \frac{1}{\zeta} A_p \hat{G}_{jt} - \frac{1}{\zeta} B_p \hat{\tau}_{jt}^k - \frac{1}{\zeta} C_p \hat{\tau}_{jt}^l - \frac{1}{\zeta} D_p \hat{\tau}_{jt}^c
\end{aligned}
$$

$$
\begin{aligned}
\hat{T}_{it}^c &= \left[\frac{s_g}{\gamma - s_c} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} A_p\right] \hat{G}_{it} + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} B_p \hat{\tau}_{it}^k \\
&\quad - \left[\frac{\tau^l}{(\gamma - s_c)(1-\tau^l)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} C_p\right] \hat{\tau}_{it}^l + \left\{1 - \left[\frac{\tau^c}{(\gamma - s_c)(1+\tau^c)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} D_p\right]\right\} \hat{\tau}_{it}^c \\
&\quad - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} G_p \hat{G}_{jt} + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} H_p \hat{\tau}_{jt}^k + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} I_p \hat{\tau}_{jt}^l + \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} J_p \hat{\tau}_{jt}^c
\end{aligned}
$$

Let $\hat{G}_{jt} = \hat{\tau}_{1t}^k = \hat{\tau}_{2t}^k = \hat{\tau}_{1t}^l = \hat{\tau}_{2t}^l = \hat{\tau}_{1t}^c = \hat{\tau}_{2t}^c = 0$. Then

$$
\frac{d\widehat{GDP}_{it}}{d\hat{G}_{it}} = \left(1 - \frac{1}{\zeta}\right) A_p \qquad \frac{d\widehat{GDP}_{jt}}{d\hat{G}_{it}} = \frac{1}{\zeta} A_p
$$

Let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}_{jt}^k = \hat{\tau}_{1t}^l = \hat{\tau}_{2t}^l = \hat{\tau}_{1t}^c = \hat{\tau}_{2t}^c = 0$. Then

$$
\frac{d\widehat{GDP}_{it}}{d\hat{T}_{it}^k} = -\frac{\left(1 - \frac{1}{\zeta}\right) B_p}{1 - \left(1 - \frac{1}{\zeta}\right) B_p} \qquad \frac{d\widehat{GDP}_{jt}}{d\hat{T}_{it}^k} = -\frac{\frac{1}{\zeta} B_p}{1 - \left(1 - \frac{1}{\zeta}\right) B_p}
$$

Let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}_{1t}^k = \hat{\tau}_{2t}^k = \hat{\tau}_{jt}^l = \hat{\tau}_{1t}^c = \hat{\tau}_{2t}^c = 0$. Then

$$
\frac{d\widehat{GDP}_{it}}{d\hat{T}_{it}^l} = -\frac{\left(1 - \frac{1}{\zeta}\right) C_p}{1 - \left(1 - \frac{1}{\zeta}\right) C_p} \qquad \frac{d\widehat{GDP}_{jt}}{d\hat{T}_{it}^l} = -\frac{\frac{1}{\zeta} C_p}{1 - \left(1 - \frac{1}{\zeta}\right) C_p}
$$

Let $\hat{G}_{1t} = \hat{G}_{2t} = \hat{\tau}^k_{1t} = \hat{\tau}^k_{2t} = \hat{\tau}^l_{1t} = \hat{\tau}^l_{2t} = \hat{\tau}^c_{jt} = 0$. Then

$$\frac{d\widehat{GDP}_{it}}{d\hat{T}^c_{it}} = -\frac{\left(1 - \frac{1}{\zeta}\right) D_p}{1 - \left[\frac{\tau^c}{(\gamma - s_c)(1+\tau^c)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} D_p\right]} \qquad \frac{d\widehat{GDP}_{jt}}{d\hat{T}^c_{it}} = -\frac{\frac{1}{\zeta} D_p}{1 - \left[\frac{\tau^c}{(\gamma - s_c)(1+\tau^c)} - \frac{\kappa+1}{(\gamma - s_c)(1-\alpha)} D_p\right]}$$

Appendix C Solving the model in Section 4

C.1 Equilibrium conditions

Replace $R^k_{i,t+1}$ by $\frac{\alpha Y_{i,t+1}}{K_{it}}$. Define $\iota_{it} \equiv \frac{\mu_{it}}{\lambda_{it}}$.

$$\theta(1+\tau^c_{it}) L^{\kappa+1}_{it} C^{\gamma}_{it} = (1-\tau^l_{it}) q^{\Lambda}_{it} (1-\alpha) Y_{it} \tag{63}$$

$$1 = \iota_{it}\left[1 - \phi\left(\frac{I_{it}}{K_{i,t-1}} - \delta\right)\right] \tag{64}$$

$$\frac{C^{-\gamma}_{it}}{1+\tau^c_{it}} = E_t \frac{\beta R_{it} C^{-\gamma}_{i,t+1}}{1+\tau^c_{i,t+1}} \tag{65}$$

$$\iota_{it}\frac{C^{-\gamma}_{it}}{1+\tau^c_{it}} = E_t \frac{\beta C^{-\gamma}_{i,t+1}}{1+\tau^c_{i,t+1}} \left\{(1-\tau^k_{i,t+1}) q^{\Lambda}_{i,t+1} \frac{\alpha Y_{i,t+1}}{K_{it}} + \iota_{i,t+1}\left[(1-\delta) + \frac{\phi}{2} I^2_{i,t+1} K^{-2}_{it} - \frac{\phi}{2}\delta^2\right]\right\} \tag{66}$$

$$C_{it} + I_{it} + G_{it} = q^{\Lambda}_{it} Y_{it} \tag{67}$$

Equations (63)-(67) along with the law of motion for capital, (8), the productivity shock processes, (11) and (12), the fiscal policy rules, (14)-(17), the government budget constraint, (43), and the production function, (49), define a competitive equilibrium.

C.2 Steady state

R^k, s_k, s_c, and θ are the same as those in B.2. We do not have s_T anymore. Instead, we have

$$s_d \equiv \frac{D}{Y} = \frac{\alpha\tau^k + (1-\alpha)\tau^l + s_c\tau^c - s_g}{1/\beta - 1}$$

In addition, $\iota = 1$ and $R = 1/\beta$.

C.3 Log-linearized system

The log-linearized system consists of equations (31) and (32), equations (52) through (59), and

$$\hat{\iota}_{it} = \phi\delta\hat{I}_{it} - \phi\delta\hat{K}_{i,t-1} \tag{68}$$

$$\gamma\hat{C}_{it} + \hat{R}_{it} + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{it} = E_t\left(\gamma\hat{C}_{i,t+1} + \frac{\tau^c}{1+\tau^c}\hat{\tau}^c_{i,t+1}\right) \tag{69}$$

$$R\left(\hat{\iota}_{it} + \hat{R}_{it}\right) = E_t\left\{\left(1-\tau^k\right)R^k\widehat{GDP}_{i,t+1} - \tau^k R^k\hat{\tau}^k_{i,t+1} + (1-\delta)\hat{\iota}_{i,t+1} + \phi\delta^2\hat{I}_{i,t+1} - \left[\left(1-\tau^k\right)R^k + \phi\delta^2\right]\hat{K}_{it}\right\} \tag{70}$$

$$s_c\hat{C}_{it} + \delta s_k\hat{I}_{it} + s_g\hat{G}_{it} = \hat{q}^{\Lambda}_{it} + \hat{Y}_{it} \tag{71}$$

$$\hat{K}_{it} = (1-\delta)\hat{K}_{i,t-1} + \delta\hat{I}_{it} \tag{72}$$

$$s_g\hat{G}_{it} + \frac{1}{\beta}s_d\hat{R}_{i,t-1} + \frac{1}{\beta}s_d\hat{D}_{i,t-1} = s_d\hat{D}_{it} + \alpha\tau^k\hat{\tau}^k_{it} + (1-\alpha)\tau^l\hat{\tau}^l_{it} + \tau^c s_c\left(\hat{\tau}^c_{it} + \hat{C}_{it}\right) + \left[\alpha\tau^k + (1-\alpha)\tau^l\right]\left(\hat{q}^{\Lambda}_{it} + \hat{Y}_{it}\right) \tag{73}$$

Appendix D Discussion of a bond economy

In this section, internationally traded bonds are added to our financial autarky model in Section 4. We consider a bond economy in which a single non-contingent bond is traded in the international asset market. The bond is denominated in terms of good X. As pointed out by Heathcote and Perri (2002), the denomination of the bond only has second order effects and hence does not influence the equilibrium allocations.

Let B_{it} denote the quantity of bonds purchased by country i residents, in period t in terms of good X. The bond pays a gross interest rate of R^b_t. The budget constraint for the representative household in country i becomes

$$(1+\tau^c_{it})C_{1t} + I_{it} + D_{it} + q^X_{it}B_{it} = (1-\tau^l_{it})q^{\Lambda}_{it}w_{it}L_{it} + (1-\tau^k_{it})q^{\Lambda}_{it}R^k_{it}K_{i,t-1} + R_{i,t-1}D_{i,t-1} + q^X_{it}R^b_{t-1}B_{i,t-1} \tag{74}$$

Compared to the financial autarky model in Section 4, we now have three more variables, R^b_t, B_{1t}, and B_{2t}. We also have three more equations to solve the bond model. The first two equations arise from households' optimization problem.

$$R_{1t}q^X_{1t} = E_t q^X_{1,t+1}R^b_t \tag{75}$$

$$R_{2t}q^X_{2t} = E_t q^X_{2,t+1}R^b_t \tag{76}$$

The third equation is the bond market clearing condition

$$B_{1t} + B_{2t} = 0 \tag{77}$$

In the bond economy model, Lemma 2 no longer holds. Trade balance in the two countries are given by

$$TB_{1t} \equiv \left(X_{2t} - \frac{q^Z_{1t}}{q^X_{1t}}Z_{1t}\right)/Y_{1t} = \left(B_{1t} - R^b_{t-1}B_{1,t-1}\right)/Y_{1t} \tag{78}$$

$$TB_{2t} \equiv \left(Z_{1t} - \frac{q_{2t}^X}{q_{2t}^Z} X_{2t} \right) / Y_{2t} = \left(\frac{q_{2t}^X}{q_{2t}^Z} B_{2t} - \frac{q_{2t}^X}{q_{2t}^Z} R_{t-1}^b B_{2,t-1} \right) / Y_{2t} \tag{79}$$

The quantities of good X and good Z in the two countries are given by

$$X_{1t} = \omega \left(q_{1t}^X \right)^{1-\eta} Y_{1t} + \omega \left(q_{1t}^X \right)^{1-\eta} R_{t-1}^b B_{1,t-1} - \omega \left(q_{1t}^X \right)^{1-\eta} B_{1t} \tag{80}$$

$$X_{2t} = (1-\omega) \left(q_{2t}^X \right)^{-\eta} q_{2t}^Z Y_{2t} + (1-\omega) \left(q_{2t}^X \right)^{1-\eta} R_{t-1}^b B_{2,t-1} - (1-\omega) \left(q_{2t}^X \right)^{1-\eta} B_{2t} \tag{81}$$

$$Z_{1t} = (1-\omega) \left(q_{1t}^Z \right)^{-\eta} q_{1t}^X Y_{1t} + (1-\omega) \left(q_{1t}^Z \right)^{-\eta} q_{1t}^X R_{t-1}^b B_{1,t-1} - (1-\omega) \left(q_{1t}^Z \right)^{-\eta} q_{1t}^X B_{1t} \tag{82}$$

$$Z_{2t} = \omega \left(q_{2t}^Z \right)^{1-\eta} Y_{2t} + \omega \left(q_{2t}^Z \right)^{-\eta} q_{2t}^X R_{t-1}^b B_{2,t-1} - \omega \left(q_{2t}^Z \right)^{-\eta} q_{2t}^X B_{2t} \tag{83}$$

However, the other lemmas still hold if we assume that net foreign asset position in steady state is zero, that is,

$$B_1 = B_2 = 0 \tag{84}$$

The log-linearized equations are mostly the same as before. Specifically, equations (29)-(32), (52)-(59), and (68)-(73) remain the same. In this way, domestic fiscal shocks still generate sizable spillovers abroad. Note that in order to make the law of motion for bonds stationary, a very small quadratic cost on bond holdings can be imposed to solve the model (see Heathcote and Perri, 2002). An earlier version of this paper includes a similar bond economy and the results are available upon request.

Appendix E Data sources and construction

The construction of tax rates on capital income, labor income, and consumption, and government spending uses data from the Bureau of Economic Analysis' NIPA. The source and methodology of processing these data are the same as in Leeper, Walker, and Yang (2010). Fiscal variables include federal and state and local governments.

The average consumption tax rate is calculated as $\tau^c = \frac{T^c}{C - T^c}$ where T^c is taxes on production and imports less property taxes. Jones's (2002) definition of average personal income tax rate is $\tau^p = \frac{IT}{W + PRI/2 + CI}$ where IT is personal current tax revenues, W is wage and salary accruals, PRT is proprietors' income and CI is capital income. Capital income is computed as the sum of rental income, corporate profits, interest income, and $PRI/2$. Then the average labor income tax rate is calculated as $\tau^l = \frac{\tau^p (W + PRI/2) + CSI}{EC + PRI/2}$ where CSI is contributions for government social insurance and EC is compensation of employees. The average capital income tax rate is computed as $\tau^k = \frac{\tau^p CI + CT + PT}{CI + PT}$ where CT is taxes on corporate income and PT is property taxes.